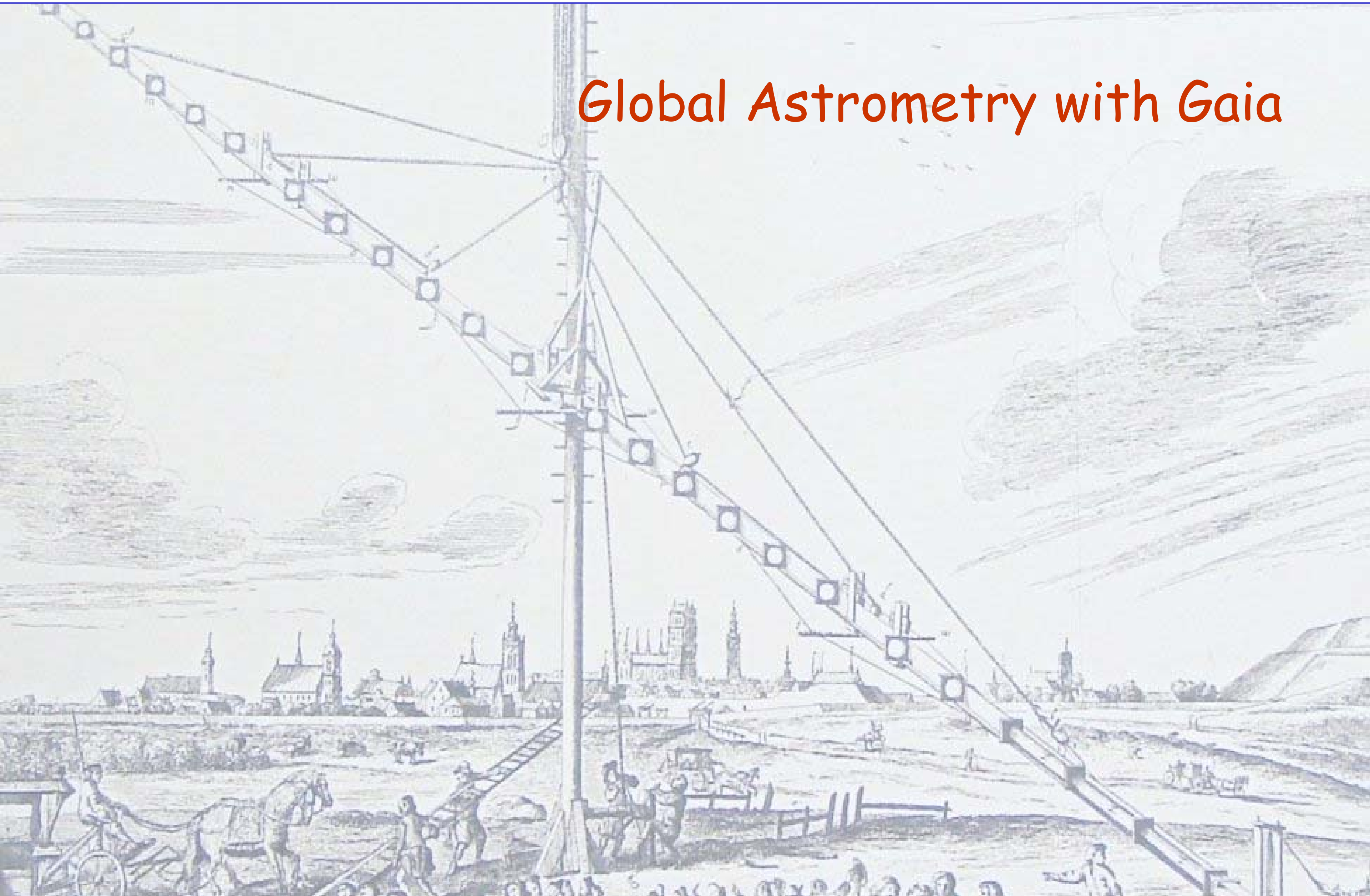
Global Astrometry with Gaia



What is *global* in global astrometry ?

- Small field astrometry can be very accurate, even more than Gaia
  - but it cannot be used to relate fields at large angular distance
- How to eliminate the systematic distortions at large angle ?
  - One needs to have direct and numerous connections between sources at large angular separations
    - in a small field the distortions are not seen, as they are constant over the frame size
    - in a very very large field, or in two connected fields, the possible distortions are different in each field and are included in the model
- To produce a full-sky frame, observations of many sources, in many directions must be processed together
  - the astrometry is then global

The Gaia conundrum

Assume your are on board of a spinning spaceship to do astrometric observations

- If one knew perfectly the rotational motion of the platform, it is possible to map the sky from local measurements

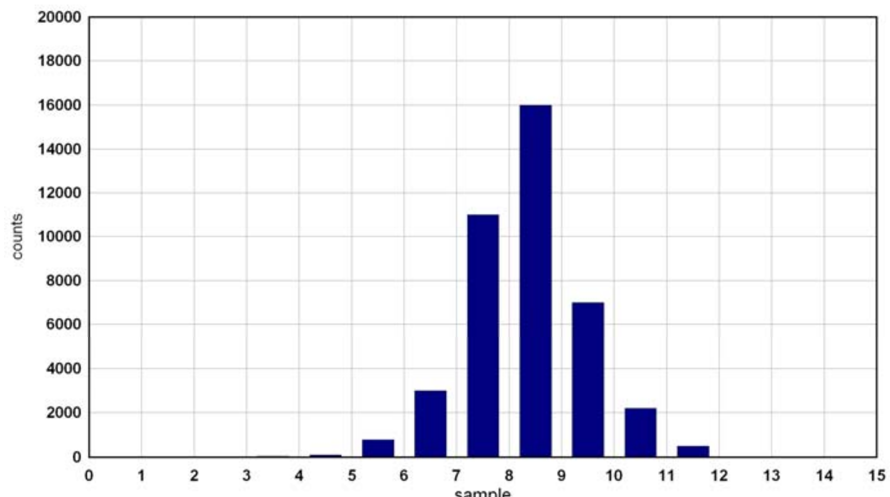
- If one knew where the stars are, it is possible to monitor the attitude of the spacecraft from local measurements

- With Gaia, one knows neither in advance and one determines both from local measurements !
    - in fact almost both, 6 frame orientation parameters are free

- Keyword : Connectivity between sources + smooth attitude

Global Iterative Solution

- Gaia observes transit times of point sources across fiducial lines on the detector
  - there are 9 observations per transit on the astrometric FOV
  - this crossing is that of the image centre relative to CCD
  - the local astrometric centring accuracy is about 200 μas at G =15
    - this is 1/300 of a pixel size along-scan
    - achievable with the ~ 40,000 counts over 4.5 s of integration
  - there are on the average 700 such measurements per star



typical image sample at G = 15

Gaia DPAC

- Central Problem:

  - For each of 10^9 observed celestial objects we want to determine six astrophysical parameters:
    - Position on celestial sphere: α,δ
    - Parallax (distance): π
    - Proper motion: μ_α, μ_δ
    - Radial velocity: v_R
  - at the μas level (π: <25μas@V=15, <7μas@V<10)
  - using (in theory) no a priori knowledge of these quantities but derive them from observation data alone in a self-consistent manner

Astrometric Core Solution

- Each observed object entering the FOV transits 9 AF CCDs
  - 9 elemental observations per object per field transit
  - one observation: a CCD centroid of the image in pixel units
- ~10^9 objects in total
- In 5 years we will have ~80 FOV transits per sources
  - 10^{11} transits
  - 10^{12} elementary data

➔ 1 trillion observations

Astrometric Core Solution

Gaia DPAC

- Need to determine > 5 x 1 billion unknowns from the 10^{12} observations using an observation model that incorporates
  - **Satellite attitude → 40 x 10^6 unknowns**
  - **Calibration parameters → 1 million unknowns**
  - **Global astrophysical parameters → ~ 10**
- Could set up a system of equations that solves directly for the unknowns – system is manifold over-determined
- Observation model not too complex
- Problem: Connectivity of the parameters

Observational equations

- The transit time of a given star can be predicted from:
  - the star astrometric parameters $\alpha, \delta, \varpi, \mu_\alpha, \mu_\delta$ **S**
  - the attitude of the satellite $q(t) : q_0, q_1, q_2, q_3$ **A**
  - the orbit of Gaia in barycentric frame $\mathbf{r}(t), \mathbf{v}(t)$
  - the imaging properties of the telescopes, the basic angle between the two fields, a scale factor to link length and angles, the geometry of the CCD mosaic **C**
  - the relationship between on-board clock ticking and TCB
  - few general parameters applicable to all stars **G**

Predicting model

- Basic direct model

$$t_k = F_{al}(S_i, A_j, C_k, G, E_t)$$

- t_k: index of observation
- F_{al}: along-scan
- S_i: source ID — $\alpha, \delta, \varpi, \mu_\alpha, \mu_\delta$
- A_j: quaternion index
- C_k: calibration data
- G: global parameters
- E_t: ephemeris

- Example (for a minor planet)

1887.6355353937 **2855 Bastian** 1 18.77 0.01609 **3** **571**

490.791788164049 42404410497373876 490.791928426528 **42404422616052008** **2015-03-03T15:15:10.258027911**

5.093826158 -0.495660889 **291.8547404** **-28.3992770** 0.38392 0.03785 -5.20 13.17 109.66 254.9

Observation equation

- Linearization about the provisional values

$$R = t_{obs} - t_{comp} = \frac{\partial F_{al}}{\partial S_i} \Delta S_i + \frac{\partial F_{al}}{\partial A_j} \Delta A_j + \frac{\partial F_{al}}{\partial C_k} \Delta C_k + \frac{\partial F_{al}}{\partial G} \Delta G + \varepsilon$$

- One solves for a subset of well-behaved stars
  - several 10^7, up to 100 millions
- Typically at mission end
  - 500 millions unknowns for stars
  - 20 to 40 millions for attitude
  - 1 million for instrument
  - 100 for general parameters
- Global problem with dense interconnections
  - direct solution not feasible with current means

Block solution

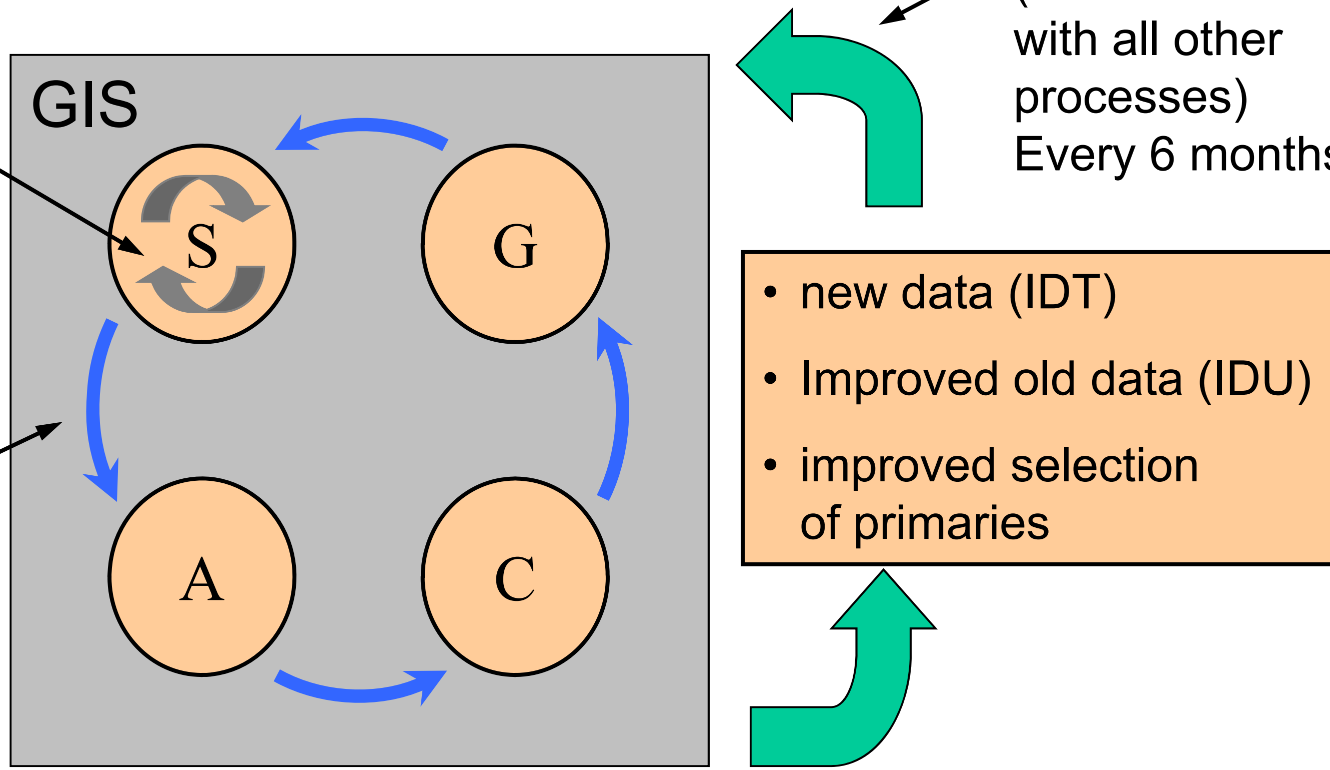
- Key: relax the connectivity issue

	known		unknown	
Full problem:	R	=	$S + A + C + G + \varepsilon$	
If one knows attitude, instrument … then	$R - A - C - G$	=	$S + \varepsilon$	10^8 small problems
If one knows stars, instrument … then	$R - S - C - G$	=	$A + \varepsilon$	10^7 small problems
	$R - S - A - G$	=	$C + \varepsilon$	
	$R - S - A - C$	=	$G + \varepsilon$	

Astrometry: Block Iterative System

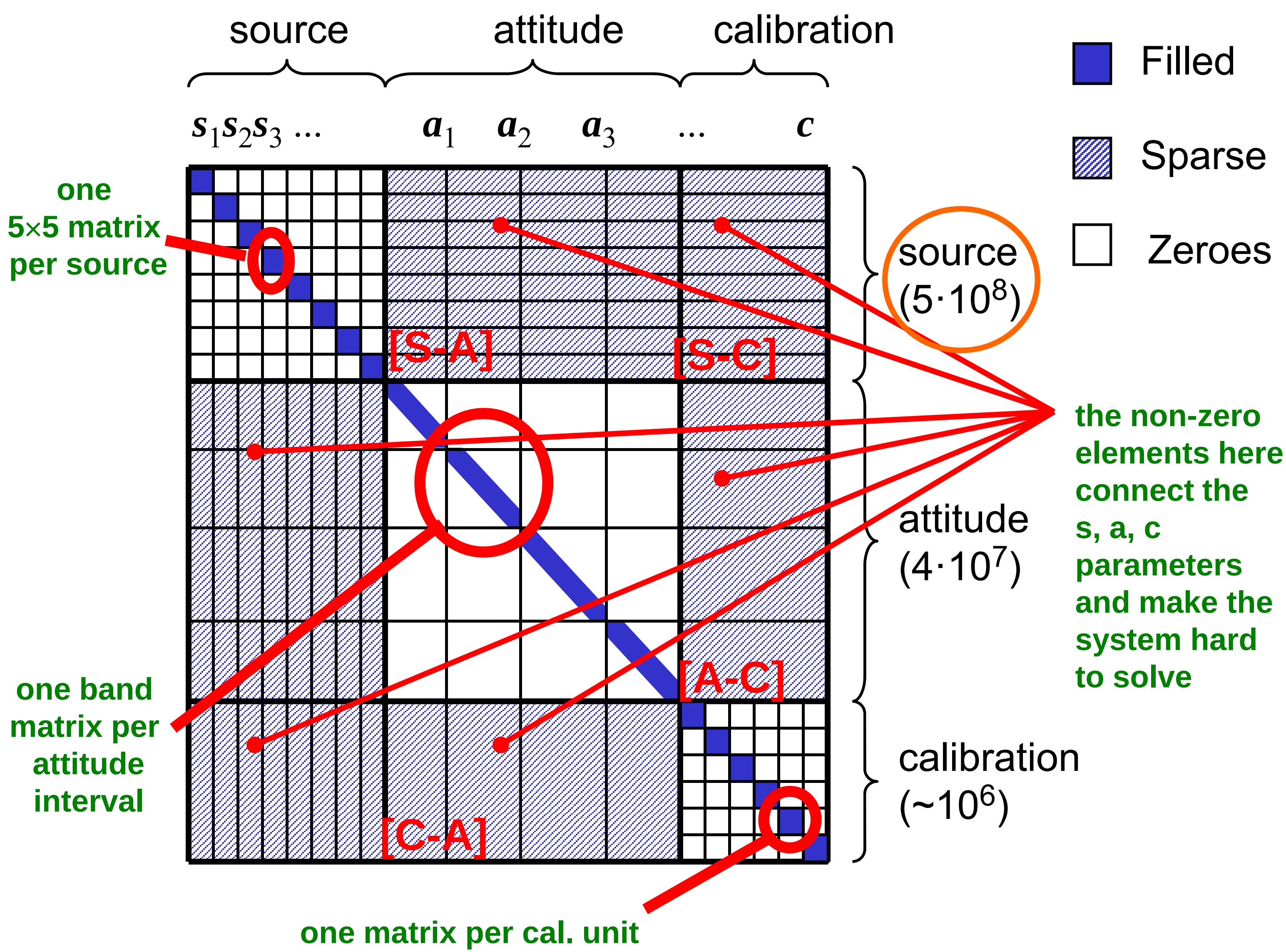
inner iteration
(non-linearity, outliers)

outer iteration
(interaction with all other processes)
Every 6 months



GIS iteration
(S-A-C-G cross-terms)

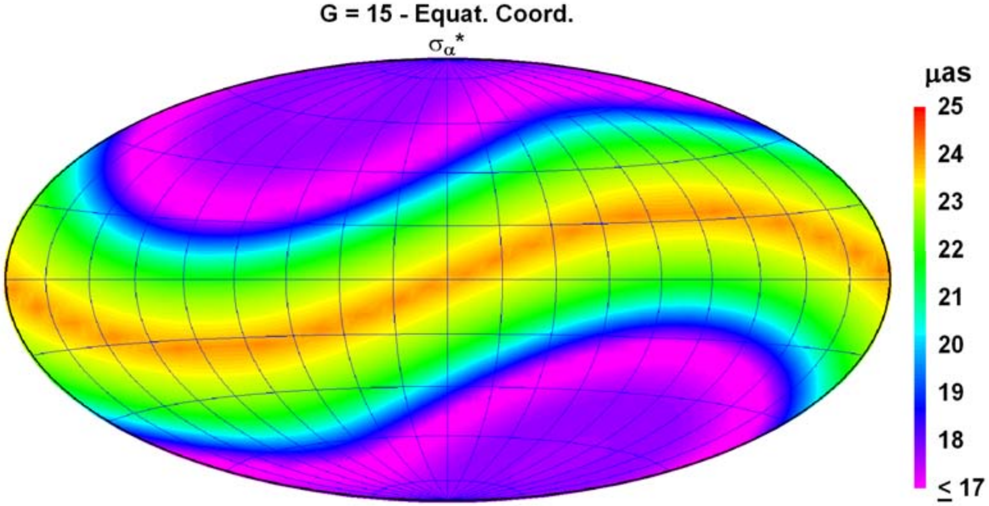
S = Sources
A = Attitude
C = Calibration
G = System, relativity



Credit: U. Lammers

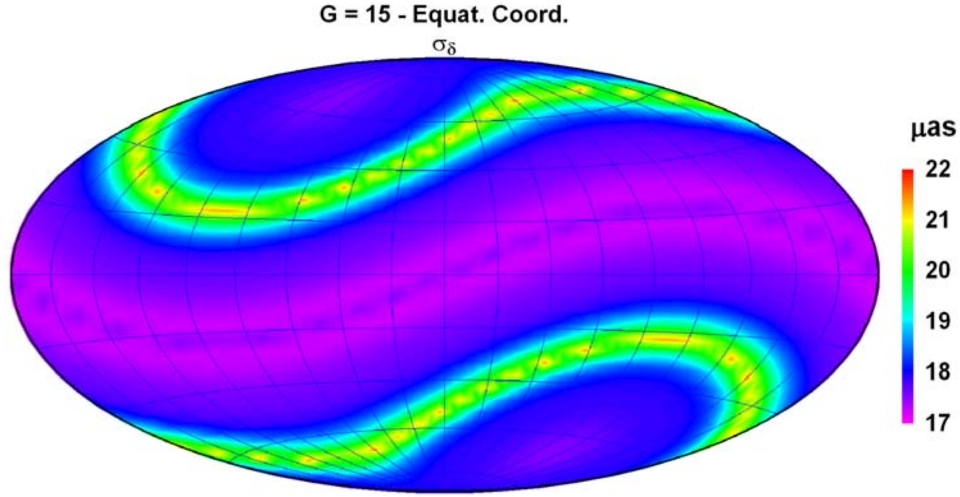
Sky distribution – Positions (~ 2016.5)

- Plots for G =15, but scalable to other magnitudes



$\sigma_\alpha^* - \mu\mathrm{as}$

$$<\sigma_\alpha^*> = 21\mu\mathrm{as}$$



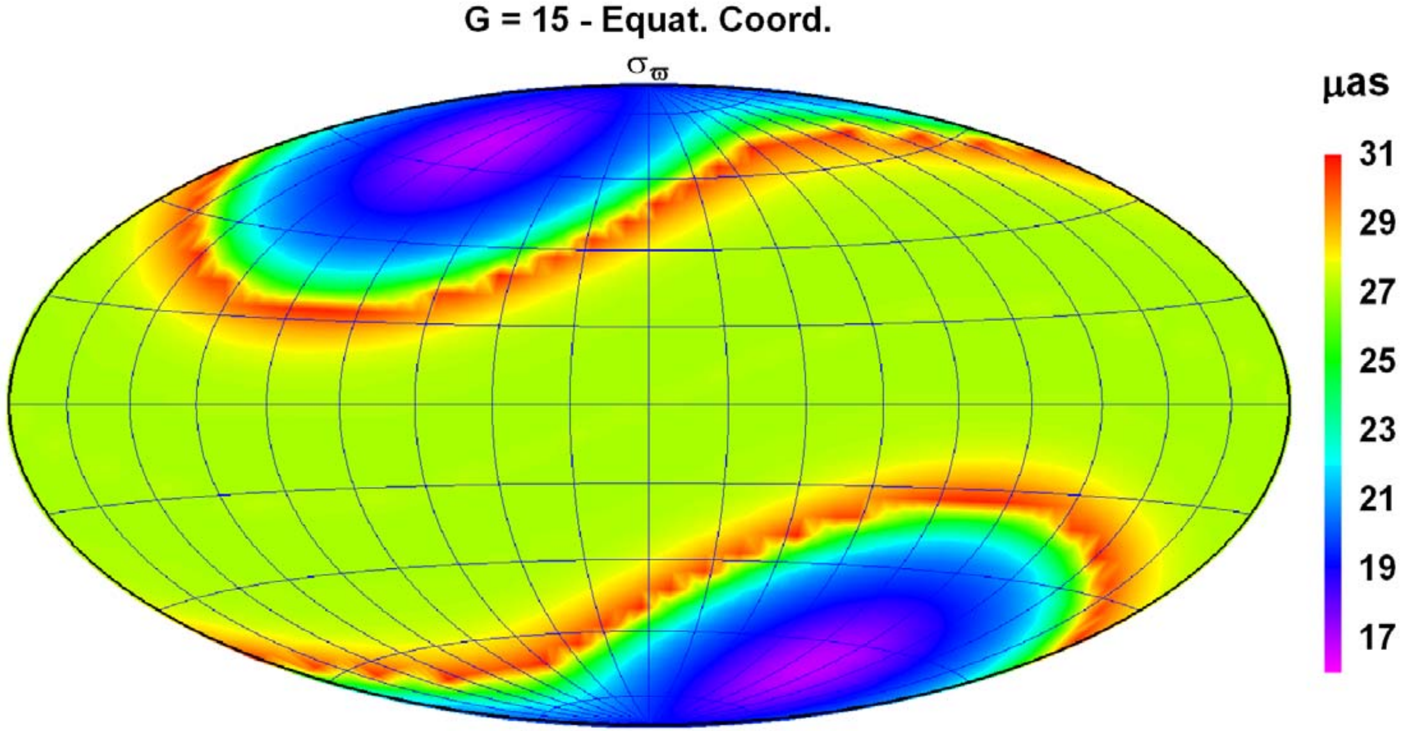
$$<\sigma_\delta> = 18\mu\mathrm{as}$$

$\sigma_\delta - \mu\mathrm{as}$

Equatorial coordinates

Sky distribution – Parallaxes

- Plot for G =15, but scalable to other magnitudes



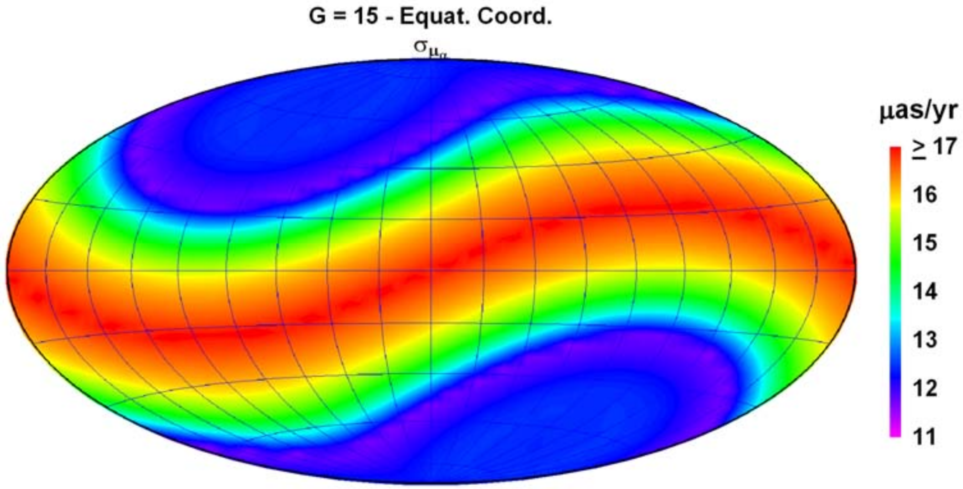
$\sigma_{\varpi} - \mu\text{as}$

$<\sigma_{\varpi}> = 25\mu\text{as}$

Equatorial coordinates

Sky distribution – Proper motions

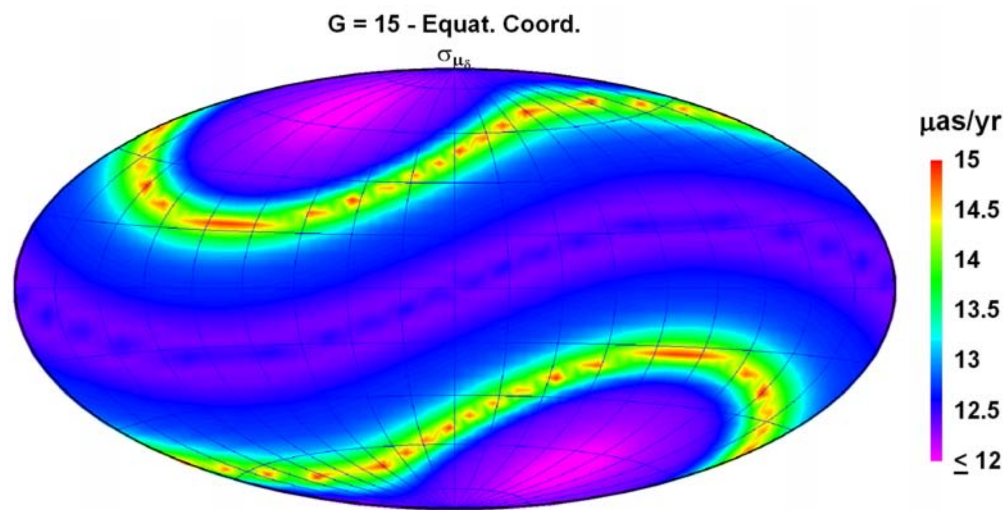
- Plots for G =15, but scalable to other magnitudes



$\sigma^*_{\mu_\alpha} - \mu\text{as yr}^{-1}$

$$<\sigma^*_{\mu_\alpha}> = 15\mu\text{as yr}^{-1}$$

$$<\sigma_{\mu_\delta}> = 13\mu\text{as yr}^{-1}$$

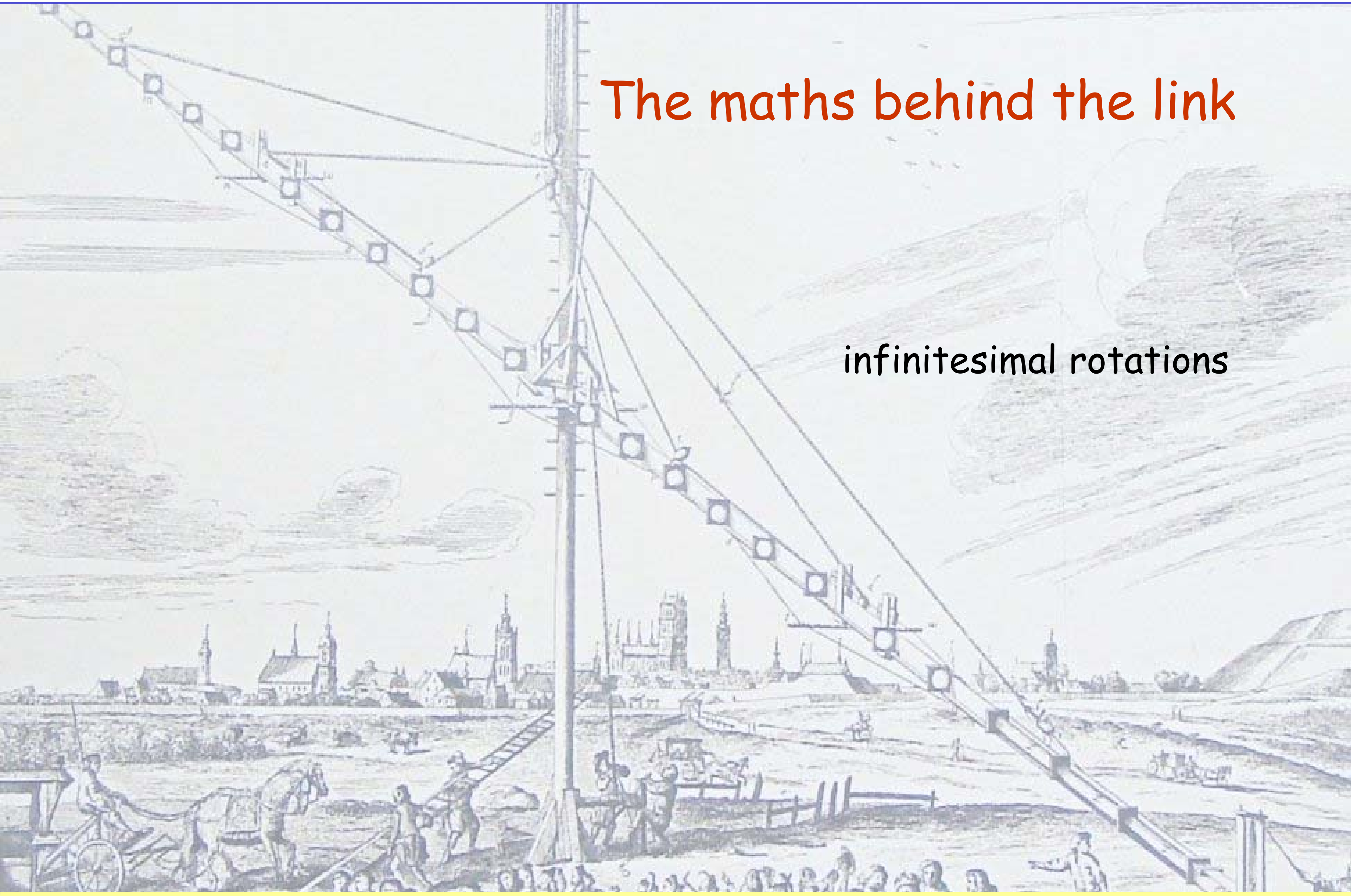


$\sigma_{\mu_\delta} - \mu\text{as yr}^{-1}$

Equatorial coordinates

The maths behind the link

infinitesimal rotations



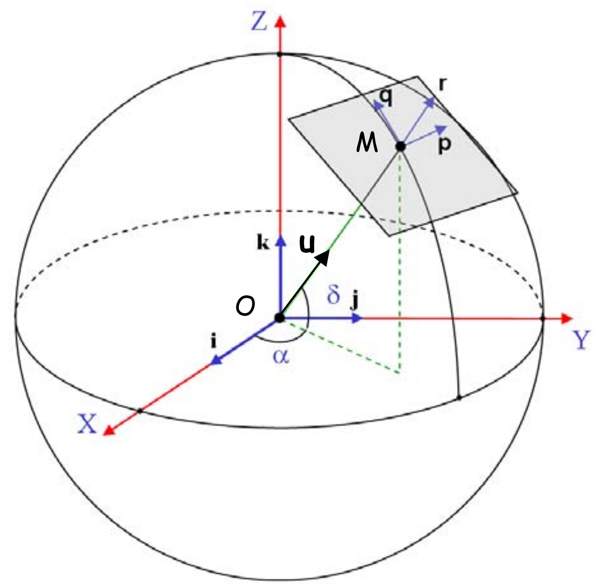
Infinitesimal rotations I

Gaia DPAC

- Local frame

$\mathbf{u}$ unit vector in direction $\mathbf{OM}$

$$\mathbf{u} = \begin{bmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{bmatrix}$$



$$\mathbf{p} = \frac{1}{\cos\delta}\frac{d\mathbf{u}}{d\alpha} = \begin{bmatrix} -\sin\alpha \\ \cos\alpha \\ 0 \end{bmatrix} \qquad \mathbf{q} = \frac{d\mathbf{u}}{d\delta} = \begin{bmatrix} -\cos\alpha\sin\delta \\ -\sin\alpha\sin\delta \\ \cos\delta \end{bmatrix}$$

$[\mathbf{p},\mathbf{q},\mathbf{r}]$ direct orthonormal triad

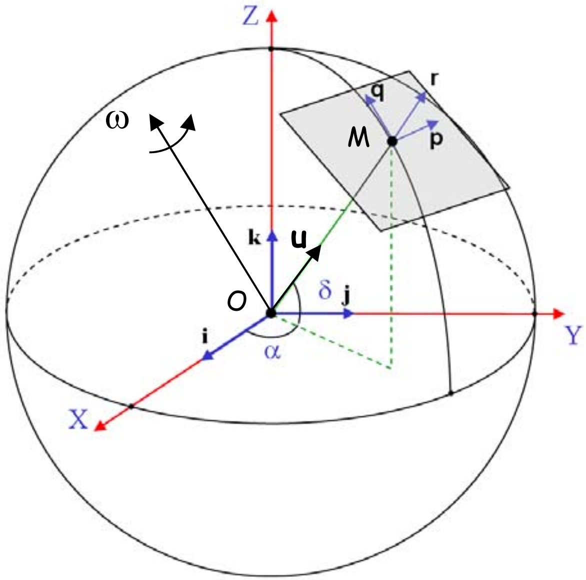
$$[\mathbf{i},\mathbf{j},\mathbf{k}] \Rightarrow [\mathbf{r},\mathbf{p},\mathbf{q}] \ : \ R_2(-\delta)R_3(\alpha)$$

$$\mathbf{V}_{[\mathbf{r},\mathbf{p},\mathbf{q}]} = \begin{bmatrix} \cos\alpha\cos\delta & \sin\alpha\cos\delta & \sin\delta \\ -\sin\alpha & \cos\alpha & 0 \\ -\cos\alpha\sin\delta & -\sin\alpha\sin\delta & \cos\delta \end{bmatrix} \mathbf{V}_{[\mathbf{i},\mathbf{j},\mathbf{k}]}$$

Infinitesimal rotations II

- **Rotation** $\boldsymbol{\omega}$ $\qquad |\boldsymbol{\omega}| << 1$

$$\boxed{d\mathbf{u} = \omega \times \mathbf{r}}$$ (active form)



fixed frame

$$d\mathbf{u} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{matrix} x \\ y \\ z \end{matrix}$$ (passive form)

local frame

$$d\mathbf{u}\cdot\mathbf{p} = (\omega\times\mathbf{r})\cdot\mathbf{p} = (\mathbf{r}\times\mathbf{p})\cdot\omega = \omega\cdot\mathbf{q}$$
$$d\mathbf{u}\cdot\mathbf{q} = (\omega\times\mathbf{r})\cdot\mathbf{q} = (\mathbf{r}\times\mathbf{q})\cdot\omega = -\omega\cdot\mathbf{p}$$

$$\boldsymbol{\omega}\cdot\mathbf{q} = \boldsymbol{\omega}\cdot\frac{d\mathbf{u}}{d\delta} = \frac{d(\boldsymbol{\omega}\cdot\mathbf{u})}{d\delta} \qquad \boldsymbol{\omega}\cdot\mathbf{p} = \boldsymbol{\omega}\cdot\frac{1}{\cos\delta}\frac{d\mathbf{u}}{d\alpha} = \frac{1}{\cos\delta}\frac{d(\boldsymbol{\omega}\cdot\mathbf{u})}{d\alpha}$$

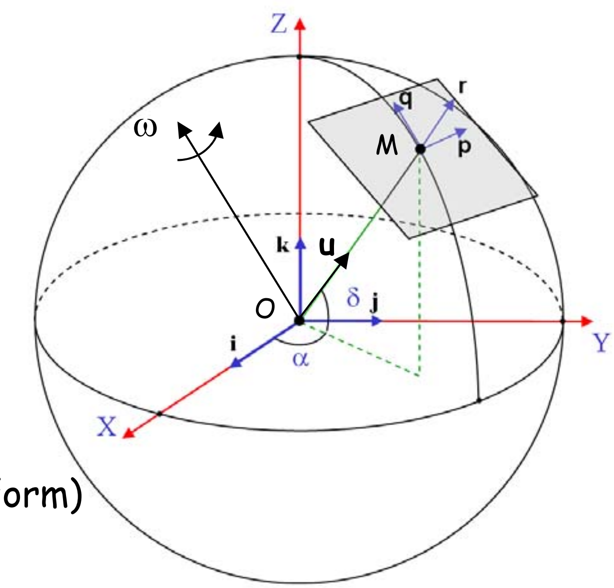
$$\boldsymbol{\omega}\cdot\mathbf{u} = \omega_x u_x + \omega_y u_y + \omega_z u_z = \omega_x\cos\alpha\cos\delta + \omega_y\sin\alpha\cos\delta + \omega_z\sin\delta$$

$$d\mathbf{u}\cdot\mathbf{p} = -\omega_x\cos\alpha\sin\delta - \omega_y\sin\alpha\sin\delta + \omega_z\cos\delta$$
$$d\mathbf{u}\cdot\mathbf{q} = \omega_x\sin\alpha - \omega_y\cos\alpha$$

Infinitesimal rotations III

$$\mathbf{du}\cdot\mathbf{p} = -\omega_x \cos\alpha \sin\delta - \omega_y \sin\alpha \sin\delta + \omega_z \cos\delta$$

$$\mathbf{du}\cdot\mathbf{q} = \omega_x \sin\alpha - \omega_y \cos\alpha$$



$$\begin{array}{c}\Delta\alpha\cos\delta \\ \Delta\delta\end{array} = \begin{bmatrix} \cos\alpha\sin\delta & \sin\alpha\sin\delta & -\cos\delta \\ -\sin\alpha & \cos\alpha & 0 \end{bmatrix}\begin{bmatrix}\omega_x \\ \omega_y \\ \omega_z\end{bmatrix}$$

(passive form)

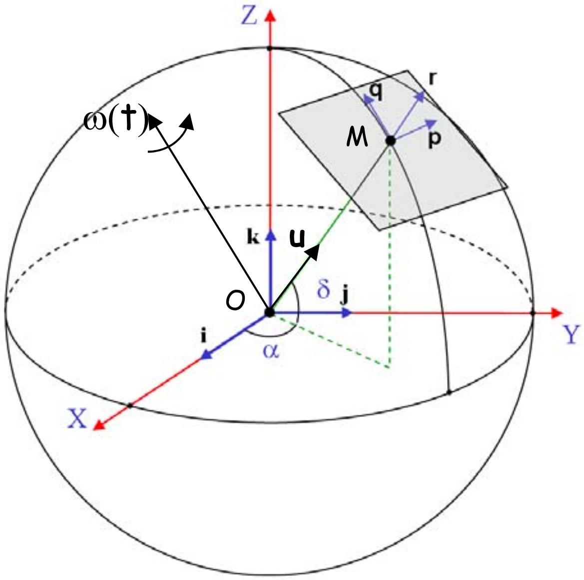
- Relationship between two frames related by a small, static rotation
  - correspondence between two catalogues of the same sources given in each frame
  - one can use the matrix to rotate one catalogue from one frame to the other
    - ω is known
  - one can use the matrix as condition equations to derive ω
    - the $(\Delta\alpha \cos\delta, \Delta\delta)_i$ are known

Infinitesimal rotations IV

- Non static form

  - rotation $\boldsymbol{\omega}(t)$ $\quad \boldsymbol{\omega}(t) = \boldsymbol{\varepsilon} + \boldsymbol{\Omega}(t - t_0)$

$$\boldsymbol{\varepsilon} : \left[\varepsilon_x, \varepsilon_y, \varepsilon_z\right]' \quad ; \quad \boldsymbol{\Omega} : \left[\Omega_x, \Omega_y, \Omega_z\right]'$$



$$\begin{matrix} \Delta\alpha^*(t) \\ \Delta\delta(t) \end{matrix} = \begin{bmatrix} \cos\alpha\sin\delta & \sin\alpha\sin\delta & -\cos\delta \\ -\sin\alpha & \cos\alpha & 0 \end{bmatrix} \begin{bmatrix} \omega_x(t) \\ \omega_y(t) \\ \omega_z(t) \end{bmatrix}$$

with two catalogue or two frames

$$\Delta\delta(t) = \delta_2(t) - \delta_1(t) \quad = \left[\delta_0 + \mu_\delta(t - t_0)\right]_2 - \left[\delta_0 + \mu_\delta(t - t_0)\right]_1 \quad \boxed{= \Delta\delta_0 + \Delta\mu_\delta(t - t_0)}$$

similarly: $\boxed{\Delta\alpha^*(t) = \Delta\alpha^*{}_0 + \Delta\mu_\alpha(t - t_0)}$

Infinitesimal rotations IV

$$\boldsymbol{\omega}(t) = \boldsymbol{\varepsilon} + \boldsymbol{\Omega}(t - t_0)$$

the transformation reads:

$$\begin{matrix} \Delta\alpha^*(t) \\ \Delta\delta(t) \end{matrix} = \begin{bmatrix} \cos\alpha\sin\delta & \sin\alpha\sin\delta & -\cos\delta \\ -\sin\alpha & \cos\alpha & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x + \Omega_x(t - t_0) \\ \varepsilon_y + \Omega_y(t - t_0) \\ \varepsilon_z + \Omega_z(t - t_0) \end{bmatrix}$$

and by identification:

$$\begin{matrix} \Delta\alpha^*_0 \\ \Delta\delta_0 \end{matrix} = \begin{bmatrix} \cos\alpha\sin\delta & \sin\alpha\sin\delta & -\cos\delta \\ -\sin\alpha & \cos\alpha & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}$$

$$\begin{matrix} \Delta\mu_a \\ \Delta\mu_\delta \end{matrix} = \begin{bmatrix} \cos\alpha\sin\delta & \sin\alpha\sin\delta & -\cos\delta \\ -\sin\alpha & \cos\alpha & 0 \end{bmatrix} \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix}$$

Application to Gaia alignment and spin I

- Two sets of coordinates of the same sources
  - one from Gaia
  - one from a reference catalogue
    - ICRF2 for the orientation, QSO catalogue for the spin
  - if the differences can be approximated by a time dependant rotation, for each source one has,

$$\Delta\alpha^*_\alpha \approx \quad \varepsilon_x \cos\alpha \sin\delta + \varepsilon_y \sin\alpha \sin\delta - \varepsilon_x \cos\delta$$

$$\Delta\delta_\delta \approx -\varepsilon_x \sin\alpha + \varepsilon_y \cos\alpha$$

$$\Delta\mu^*_\alpha \approx \quad \Omega_x \cos\alpha \sin\delta + \Omega_y \sin\alpha \sin\delta - \Omega_x \cos\delta$$

$$\Delta\mu_\delta \approx -\Omega_x \sin\alpha + \Omega_y \cos\alpha$$

- Solution for Ω and ε with least-squares

Application to Gaia alignment and spin II

- The normal matrix depends only on the source distribution and accuracy (~ magnitude)

$$N_{xx} = \sum W_\alpha \cos^2\alpha \sin^2\delta + \sum W_\delta \sin^2\alpha \qquad N_{xy} = \sum W_\alpha \sin\alpha\cos\alpha\sin^2\delta - \sum W_\delta \sin\alpha\cos\alpha$$

$$N_{yy} = \sum W_\alpha \sin^2\alpha \sin^2\delta + \sum W_\delta \cos^2\alpha \qquad N_{yz} = -\sum W_\alpha \sin\alpha\cos\delta\sin\delta$$

$$N_{zz} = \sum W_\alpha \cos^2\delta \qquad N_{zx} = -\sum W_\alpha \cos\alpha\cos\delta\sin\delta$$

- With a relatively uniform distribution in each bin of magnitude

$$<\cos^2\alpha> \;\approx\; <\sin^2\alpha> \;\approx\; 1/2$$

$$<\sin^2\delta> \approx 1/3 \qquad <\cos^2\delta> \approx 2/3$$

$$<\sin\alpha> \approx <\cos\alpha> \approx <\sin\alpha\cos\alpha> \approx 0$$

  - the normal matrix is nearly diagonal

$$\mathrm{cov}(\boldsymbol{\varepsilon}) = \mathbf{N}^{-1} \qquad \mathrm{cov}(\Omega) = \mathbf{N}^{-1}$$

alignment sources — spin sources

Offset between radio and optics

- Light and radio emission centers could be different
  - there is at the moment no real evidence below ~ 10 mas level
  - physically with QSO models one has good reasons to assume there is an offset
    - how big is an open question

model without offset

$$\begin{bmatrix} \Delta\alpha^*_0 \\ \Delta\delta_0 \end{bmatrix} = \begin{bmatrix} \cos\alpha\sin\delta & \sin\alpha\sin\delta & -\cos\delta \\ -\sin\alpha & \cos\alpha & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} + \begin{bmatrix} N_\alpha \\ N_\delta \end{bmatrix}$$

(noise)

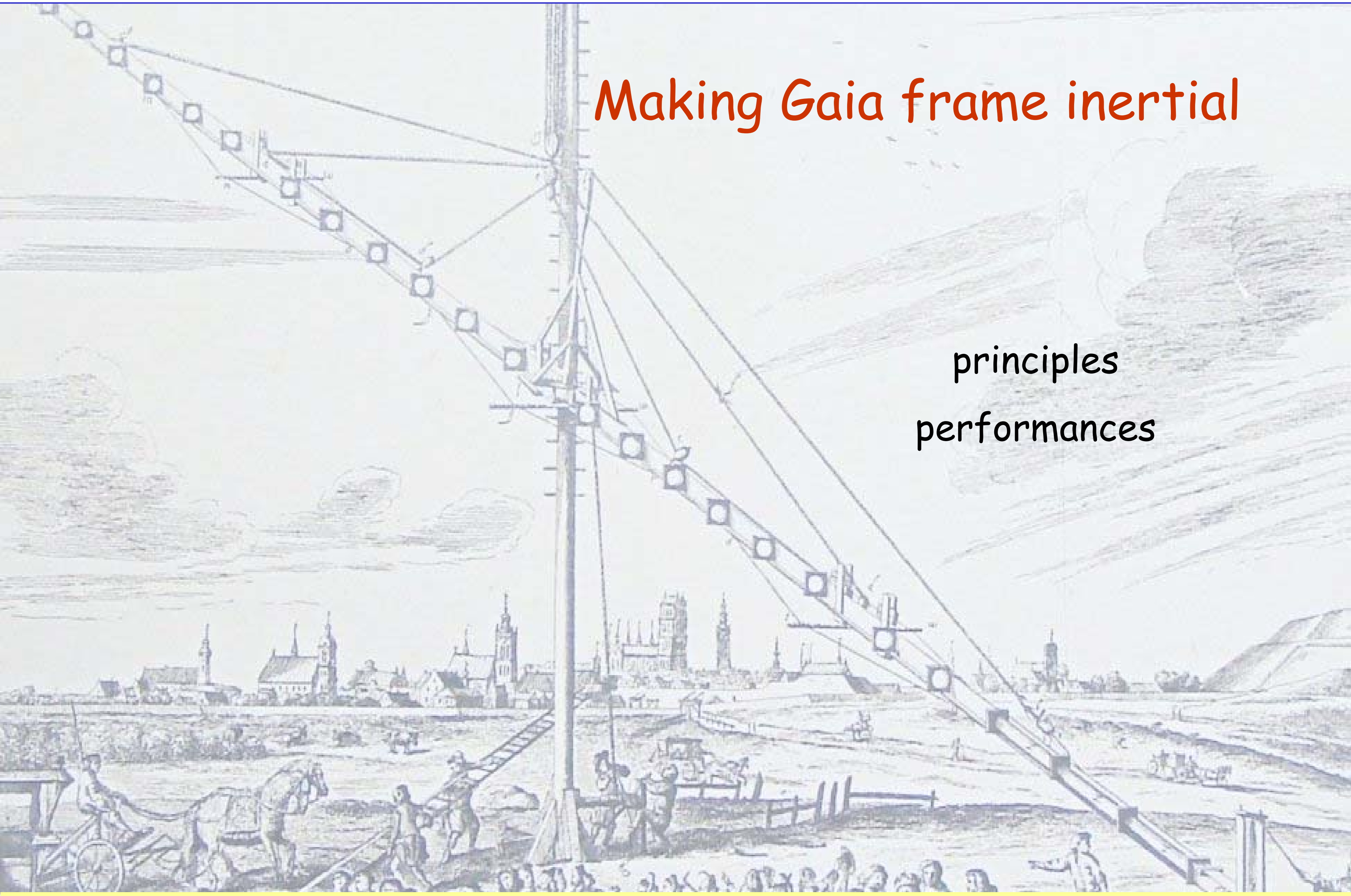
model with offset

$$\begin{bmatrix} \Delta\alpha^*_0 \\ \Delta\delta_0 \end{bmatrix} = \begin{bmatrix} \cos\alpha\sin\delta & \sin\alpha\sin\delta & -\cos\delta \\ -\sin\alpha & \cos\alpha & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} + \begin{bmatrix} O_\alpha \\ O_\delta \end{bmatrix} + \begin{bmatrix} N_\alpha \\ N_\delta \end{bmatrix}$$

(offset) (noise)

- The offset vector **O** is probably of random nature from source to source

Making Gaia frame inertial



principles

performances

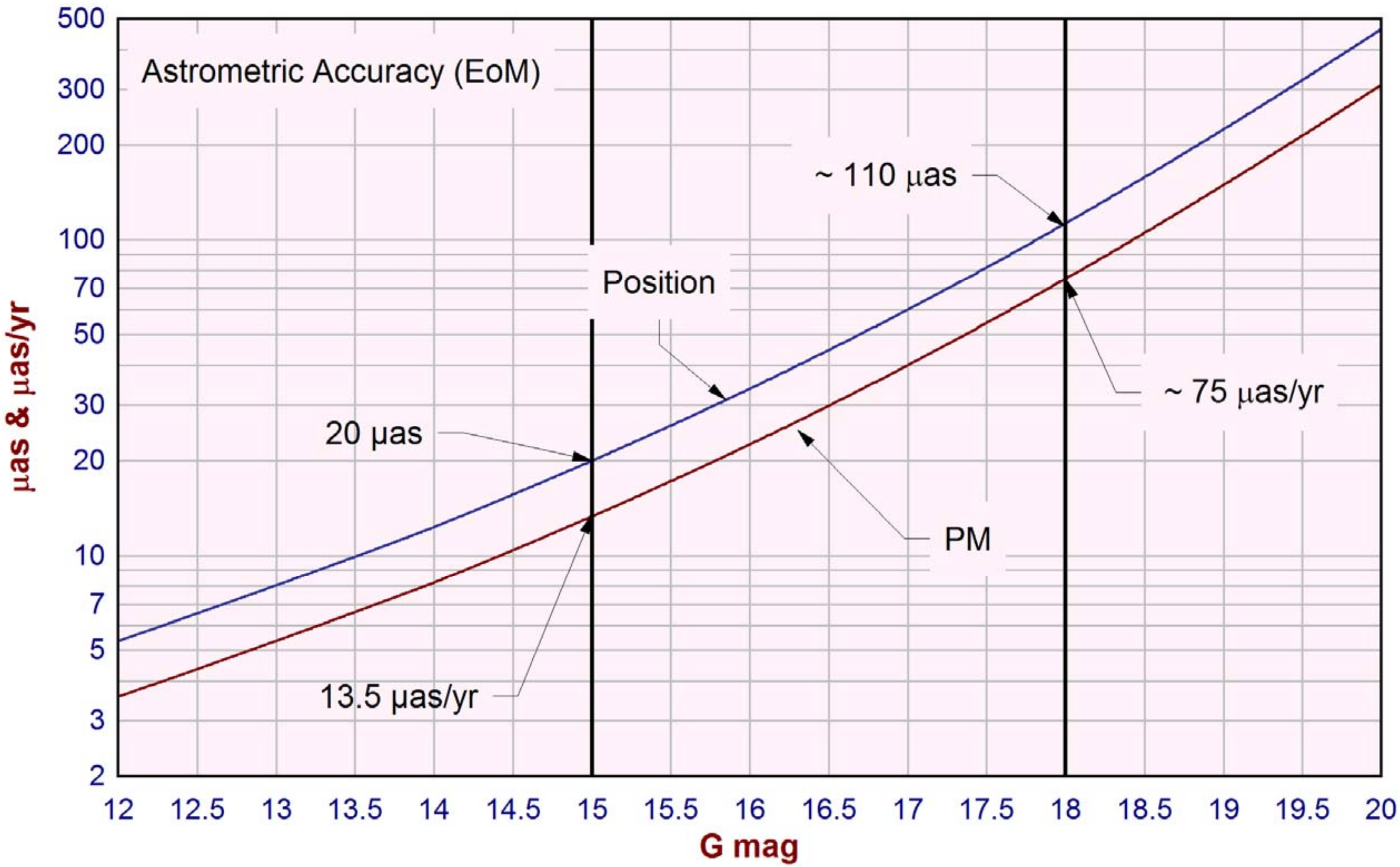
Performance on the spin determination

- Assumption
  - the only source of transverse motion in the Gaia solution comes from the free spin
- Material
  - LQAC known sources (150,000 sources $G< 20$)
  - Gaia simulated QSO catalogue (550,000 sources $G < 20$)
- Model fitting
  - A global spin ω (ω_x ω_y ω_z) on the QSO proper motions of the Gaia unrotated frame
  - the covariance matrix is computed for each bin of magnitude
- Application
  - all proper motion (stars, QSOs) are corrected for this spin pattern

Transverse motions

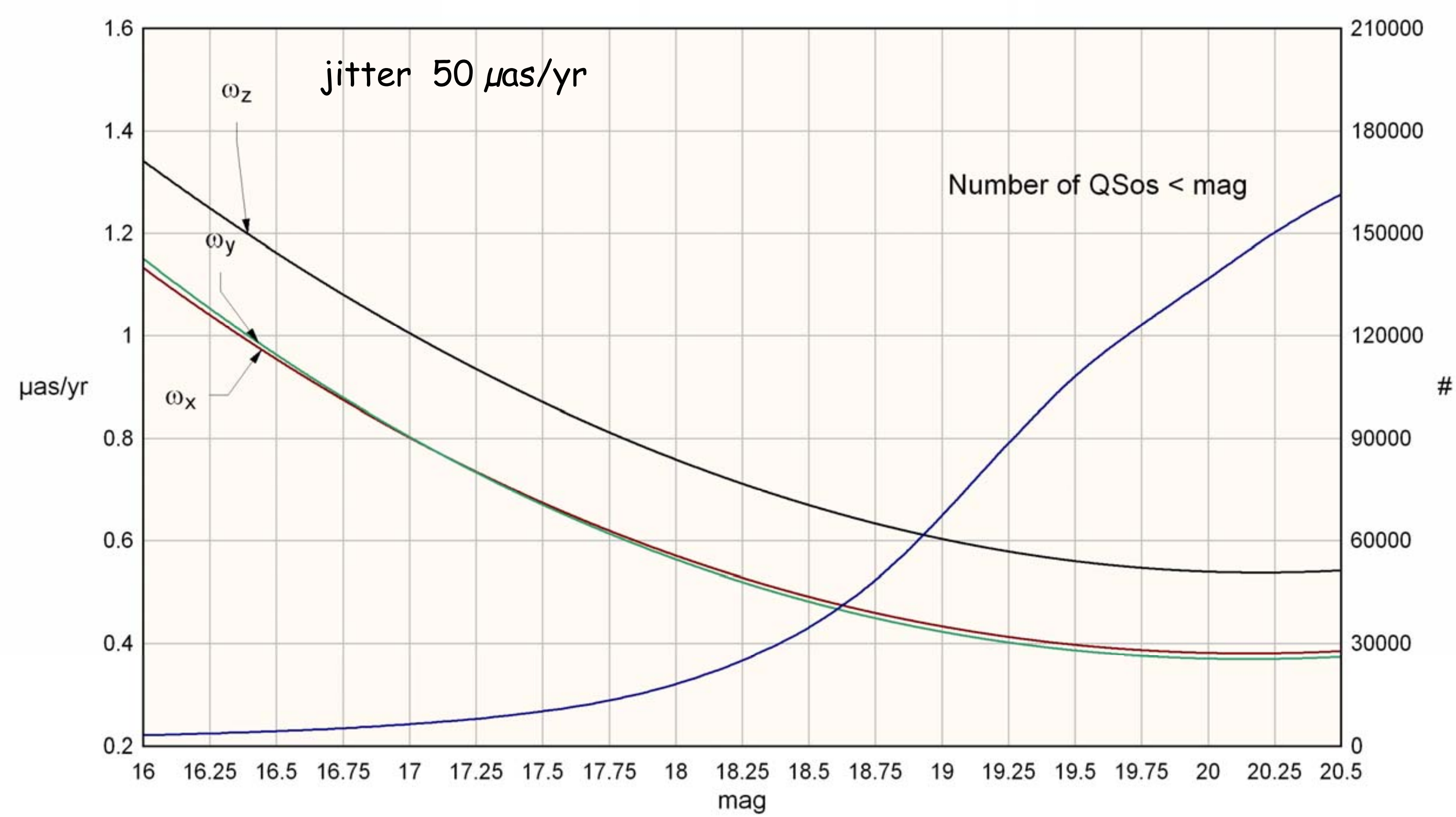
- So far no systematic transverse motion detected
  - QSOs have fixed comoving coordinates
- If $V_t \sim H_0 D$ ➔ $\mu \sim 10\ \mu as/yr$
  - VLBI in 20 yrs with $\sigma_{pos} \sim 1$ mas ➔ $\mu < 50\ \mu as$
  - but sub-mas structure instabilities (P. Charlot, 2003)
- Other sources :
  - microlensing $P = 10^{-6}$ (Belokurov) ➔ only a handful
  - matter ejection, superluminous motion
  - Variable galactic aberration
  - Macrolensing $P = 10^{-2}$ (Mignard, 2003)➔ long timescale
  - Accelerated motion in the local group
  - binary QSOs ?

Astrometric Accuracy: QSOs



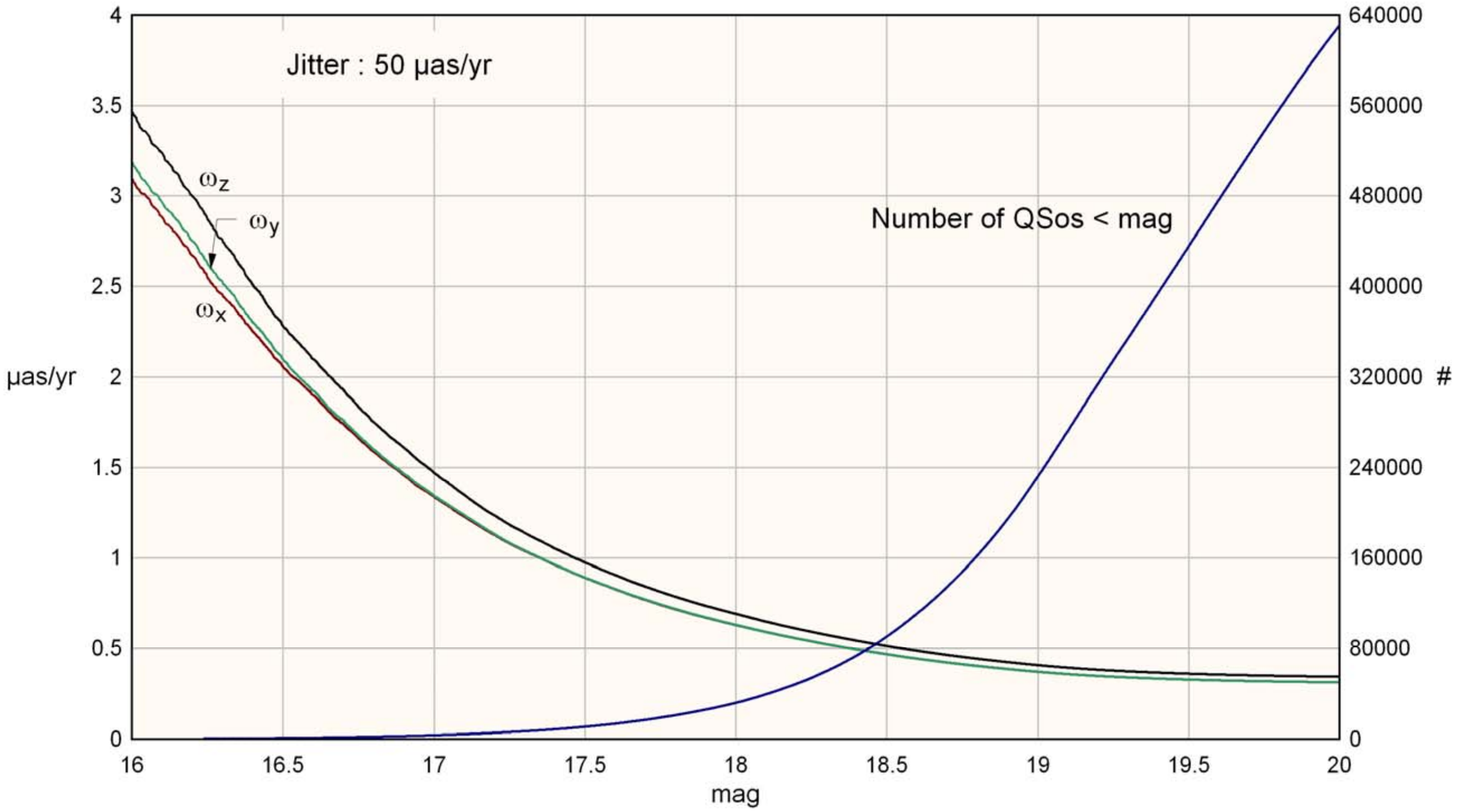
Performance with LQAC

- Spin covariance matrix computed when QSOs are constrained to have no overall motion
- The plot shows the standard error in ω



Performance with simulated QSO catalogue

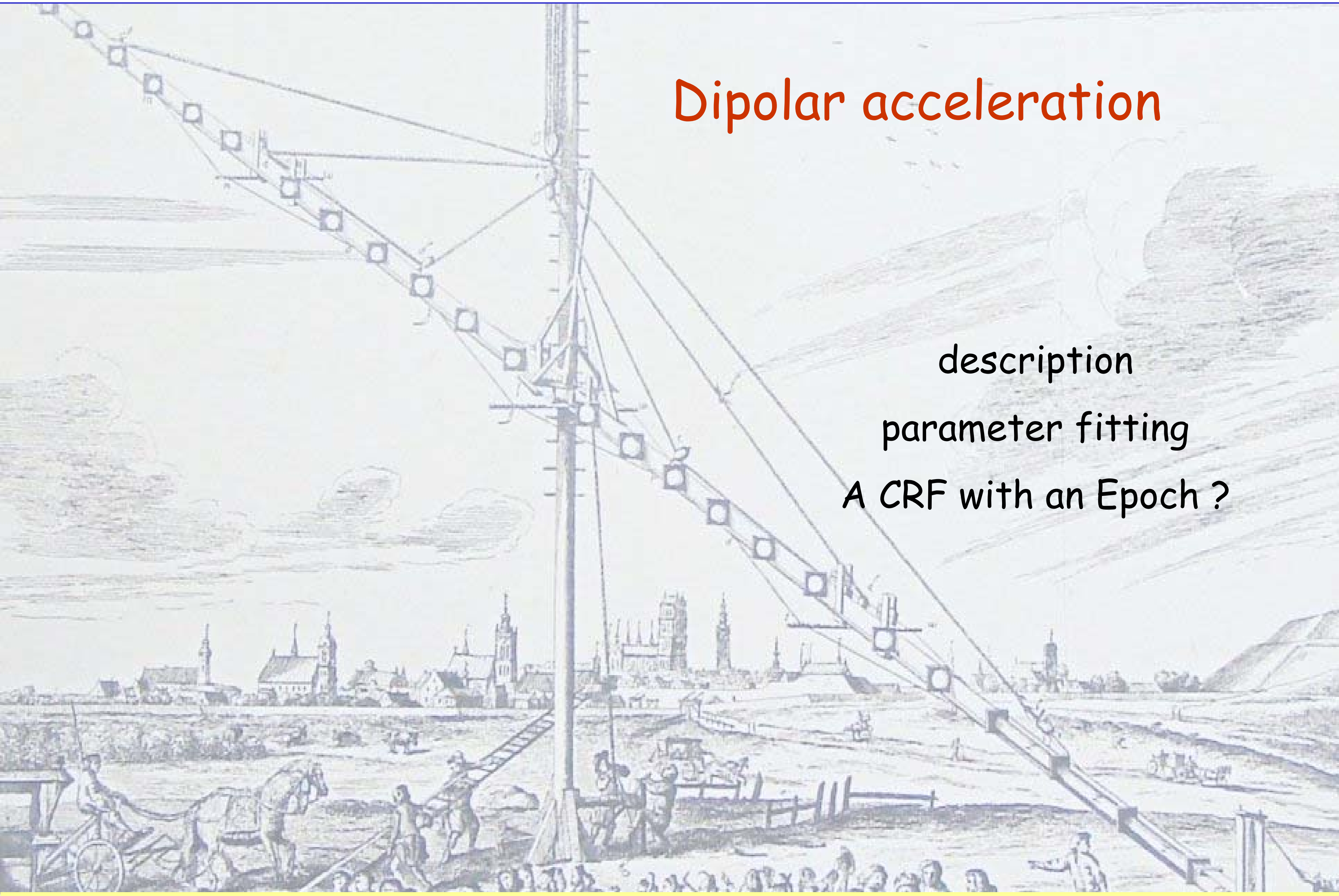
- Spin covariance matrix computed when QSOs are constrained to have no overall motion
- The plot shows the standard error in ω



Quite Challenging

We aim at a final result on the reference frame and the acceleration at 0.3 μas/yr level:

This is 1/1000 of the astrometric accuracy of the faintest sources



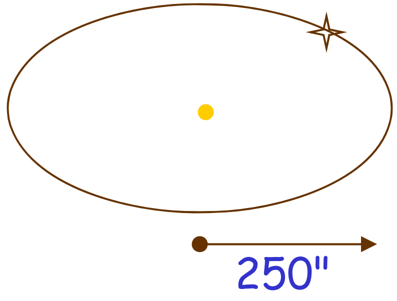
Dipolar acceleration

description

parameter fitting

A CRF with an Epoch ?

- The solar system is in motion in the Galaxy, $V \sim 220$ km s^{-1}
  - constant aberration of ~ 250" for the QSO wrt to comoving frame
  - not detectable (principle of relativity)
  - $\delta\mathbf{u} = \mathbf{v}/c$



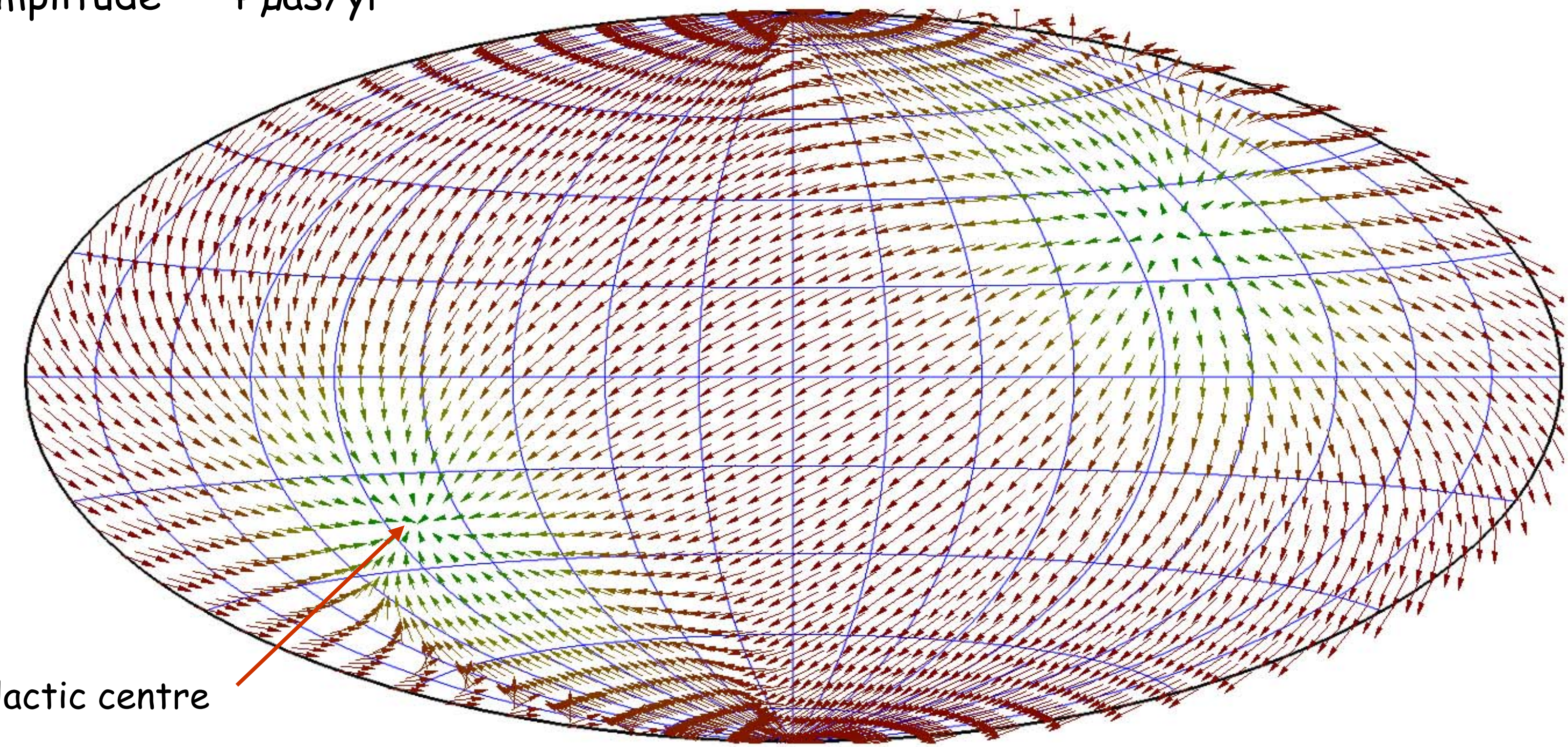
Galactic aberration over 250×10^6 years

- **But the solar motion is not uniform**
  - ~ circular motion
    - radius R ~ 8.5 kpc and period 250×10^6 yrs
  - the aberration is then variable
    - one sees a very small arc on the aberration ellipse

$$\delta\mu = \frac{d(\delta\mathbf{u})}{dt} = \frac{\mathbf{\Gamma}}{c} = \frac{V^2}{cR} \approx 4\,\mu as / yr$$

Secular Drift on QSOs

- Plot in equatorial coordinates
  - amplitude ~ 4 μas/yr



galactic centre

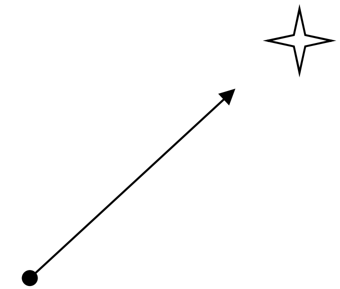
$$\mu_\alpha \cos\delta = -\frac{\Gamma_x}{c}\sin\alpha + \frac{\Gamma_y}{c}\cos\alpha$$

$$\mu_\delta = -\frac{\Gamma_x}{c}\sin\delta\cos\alpha - \frac{\Gamma_y}{c}\sin\delta\sin\alpha + \frac{\Gamma_z}{c}\cos\delta$$

General Case : detail

- For any acceleration of the SS wrt Quasars :

$$\frac{d\mathbf{u}}{dt} = \frac{\mathbf{\Gamma}}{c} - (\frac{\mathbf{\Gamma}}{c}\cdot\mathbf{u})\mathbf{u}$$



Observations

$$\mu_\alpha \cos\delta = -\frac{\Gamma_x}{c}\sin\alpha + \frac{\Gamma_y}{c}\cos\alpha$$

$$\mu_\delta = -\frac{\Gamma_x}{c}\sin\delta\cos\alpha - \frac{\Gamma_y}{c}\sin\delta\sin\alpha + \frac{\Gamma_z}{c}\cos\delta$$

- Equations similar to global rotation.
- Precision of ~ 0.3 μas/an (2 prad/yr) on Γ/c

  $= 0.2\times 10^{-10}$ m s^{-2} (γ Pionner/40)
- Galactic rotation (μ ~ 4 μas/yr)
- Acceleration of the Local Group➔ CDM ?

Rotation vs. Acceleration

- The two fields are globally orthogonal on the sphere

$$\boldsymbol{\mu}_{\mathrm{a}} = \mathbf{u} \times (\mathbf{a} \times \mathbf{u}) = \mathbf{a} - (\mathbf{a} \cdot \mathbf{u})\mathbf{u}$$

$$\boldsymbol{\mu}_{\omega} = \boldsymbol{\Omega} \times \mathbf{u}$$

$$\boldsymbol{\mu}_{\mathrm{a}} \cdot \boldsymbol{\mu}_{\omega} = (\boldsymbol{\Omega} \times \mathbf{u}) \cdot (\mathbf{a} - (\mathbf{a} \cdot \mathbf{u})\mathbf{u}) = (\boldsymbol{\Omega} \times \mathbf{u}) \cdot \mathbf{a} = (\mathbf{a} \times \boldsymbol{\Omega}) \cdot \mathbf{u}$$

  - If $\mathbf{a}$ and $\boldsymbol{\Omega}$ are parallel, then local orthogonality
  - Otherwise

$$\int_S \boldsymbol{\mu}_{\mathrm{a}} \cdot \boldsymbol{\mu}_{\omega} dS = \int_S (\mathbf{a} \times \boldsymbol{\Omega}) \cdot \mathbf{u} \; dS = (\mathbf{a} \times \boldsymbol{\Omega}) \cdot \int_S \mathbf{u} \; dS \equiv 0$$

- Orhogonal only on the average on the sphere
  - therefore, one must solve simultaneously for both vectors

Solution with dipole acceleration

- The galactic acceleration entails a systematic transverse motion of the QSOs

$$\frac{d\mathbf{u}}{dt} = \boldsymbol{\mu} = \mathbf{u} \times \left(\frac{\boldsymbol{\Gamma}}{c} \times \mathbf{u} \right) = \frac{\boldsymbol{\Gamma}}{c} - \left(\frac{\boldsymbol{\Gamma}}{c} \cdot \mathbf{u} \right) \mathbf{u}$$

$$\boldsymbol{\mu} \cdot \mathbf{p} = \frac{\boldsymbol{\Gamma} \cdot \mathbf{p}}{c} \qquad \boldsymbol{\mu} \cdot \mathbf{q} = \frac{\boldsymbol{\Gamma} \cdot \mathbf{q}}{c} \quad \text{since} \quad \mathbf{u} \cdot \mathbf{p} = \mathbf{u} \cdot \mathbf{q} \equiv 0$$

noise

with $\mathbf{a} = \frac{\boldsymbol{\Gamma}}{c}$:

$$\begin{bmatrix} \Delta\mu^*_a \\ \Delta\mu_\delta \end{bmatrix} = \begin{bmatrix} \cos\alpha\sin\delta & \sin\alpha\sin\delta & -\cos\delta \\ -\sin\alpha & \cos\alpha & 0 \end{bmatrix} \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} + \begin{bmatrix} -\sin\alpha & \cos\alpha & 0 \\ -\cos\alpha\sin\delta & -\sin\alpha\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} N_{\mu_\alpha} \\ N_{\mu_\delta} \end{bmatrix}$$

- Given the near perfect orthogonality the normal matrix is bloc diagonal

$$\begin{bmatrix} \mathbf{N} & \mathbf{0} \\ \mathbf{0} & \mathbf{N} \end{bmatrix} \qquad \text{and } \operatorname{cov}(\boldsymbol{\Omega}) \sim \operatorname{cov}(\mathbf{a}) = \mathbf{N}^{-1}$$

More general patterns on proper motions

- Proper motions seen as a vector field on S_2
- Applicable to stars and QSOs

- Expansion in Vector Spherical Harmonics T_{lm} , S_{lm}

$$\mathbf{V}(\alpha,\delta) = V_\alpha \mathbf{e}_\alpha + V_\delta \mathbf{e}_\delta = \sum_{l=1}^{l=L} \sum_{m=-l}^{m=l} (t_{lm}\mathbf{T}_{lm} + s_{lm}\mathbf{S}_{lm})$$

$l = 1 - \mathbf{S}_{1m}$	Global rotation
$l = 1 - \mathbf{T}_{1m}$	Solar system acceleration
$l > 1 - \mathbf{S}_{1m}\ \&\ \mathbf{T}_{1m}$	Stochastic field of GW

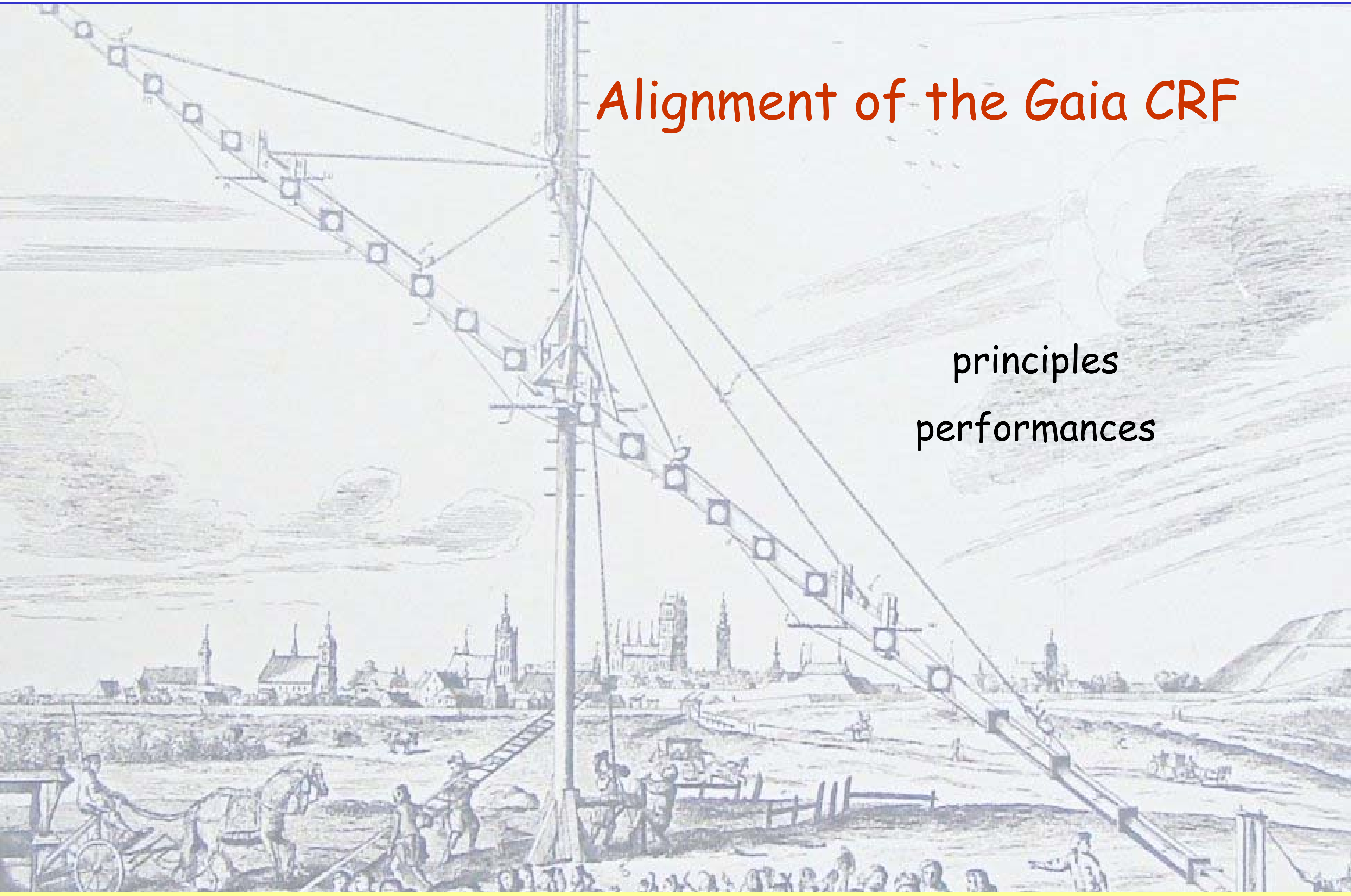
Will the Gaia-CRF have an Epoch ?

QSO Proper Motion and Epoch

- The radio ICRF is not associated to an epoch
  - defining QSOs have fixed celestial coordinates
    - they are not epoch dependant
  - the define axis direction 'for ever'
  - time is no involved in the process
- A stellar reference frame is defined at a particular epoch
  - defining stars coordinates come with their proper motion
  - the PMs are part of the fundamental catalogue
  - each star comes with a particular PM and its uncertainty
    - with N stars, there are 2N parameters needed
  - the system degrades due to the limited uncertainty of the PMs
    - accuracy in position and annual PM are similar

QSO Proper Motion and Epoch

- What about the Gaia-CRF
  - QSOs have a systematic proper motion of ~ 4 muas/yr
  - But these are not individual PM, but the result of a systematic pattern
    - only 3 parameters are required to maintain the system
    - the accuracy should be < 0.5 muas/yr
  - Individual positions of the primary sources will have an accuracy of ~ 80 muas
  - degradation will be very very slow
- Therefore : the Gaia-CRF will have an epoch attached to it
  - but it has very different meaning as for a stellar reference frame
- How to avoid it: take the origin at the galactic centre !
  - this is for the future



Alignment of the Gaia CRF

principles

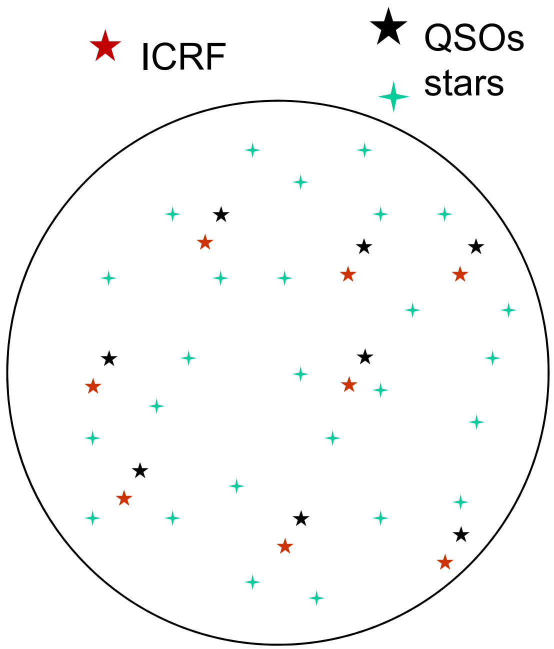
performances

Gaia alignment to ICRF

- Orientation is performed by minimizing the distances between Gaia positions and ICRF positions of common sources
  - GCRF needs to be aligned to ICRF
  - we have one infinitesimal rotations to fit$(\varepsilon_x, \varepsilon_y, \varepsilon_z)$

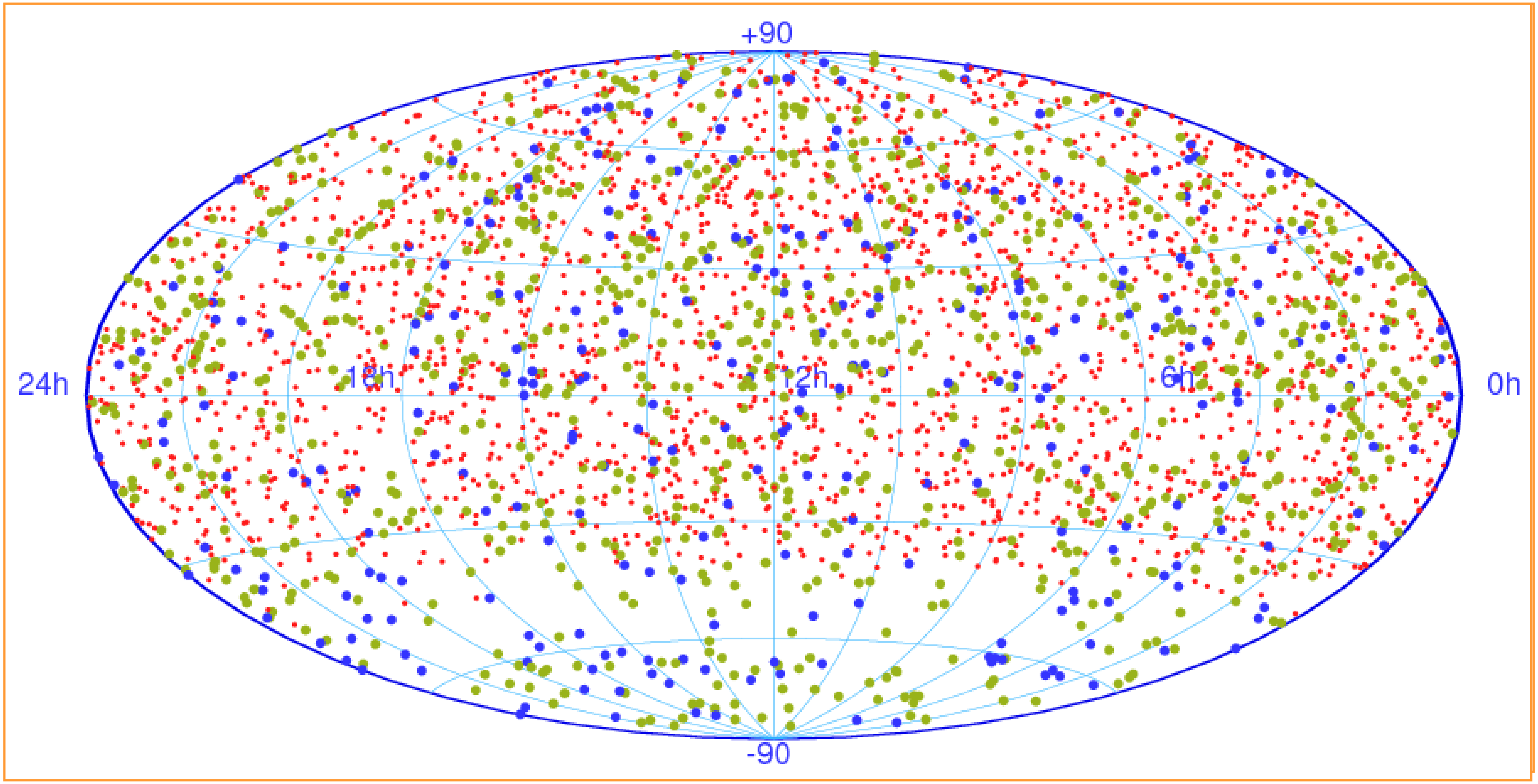
- ICRF sources are observed by Gaia
  - ~ 1500 G < 20 , 200 G < 18 - σ_{Gaia} < 100 μas
  - GCRF can be aligned to QSOs by a rotation
  - accuracy

$$\sigma_{\text{align}} \approx \frac{\sqrt{\sigma^2_{\text{Gaia}} + \sigma^2_{\text{ICRF}}}}{\sqrt{N_{\text{QSO}}}} < 10\mu as$$



ICRF-2 (2009)

σ~ 50 to 150 μas σ~ 0.2 to 2 mas σ~ 0.5 to 10 mas



• defining (294) • VLBI (923) • VLBA Calib. (2197)

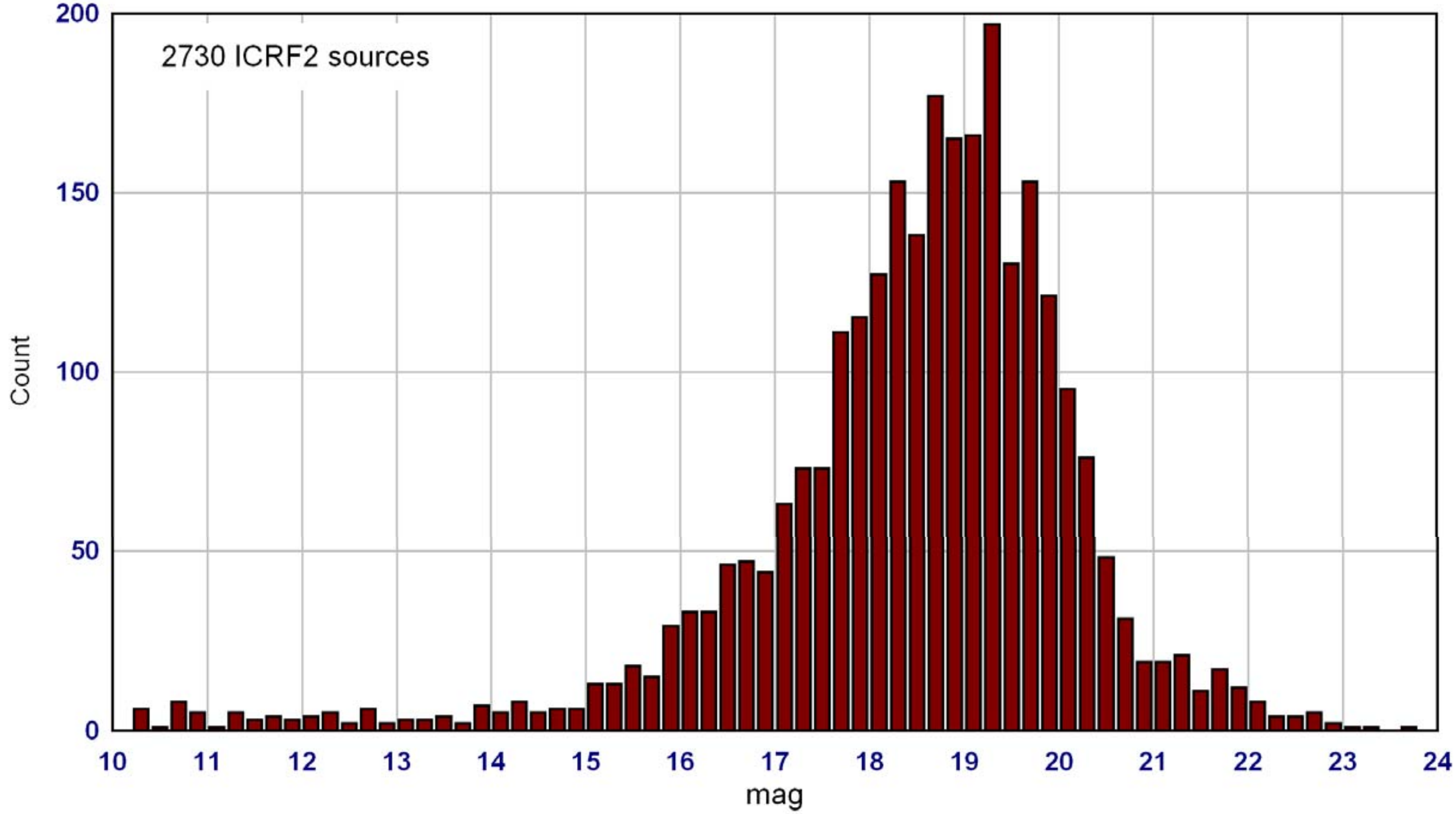
ICRF-2 in optics

- Relatively faint sources in the visible
  - From ICRF1 B Magnitude given by Veron & Veron-Cetty
    - V ~ 17-21
- No visual magnitude available in the ICRF2 publication
- Cross-matched of the LQAC with ICRF2
  - For each entry there are between 0 and 7 photometric bands available

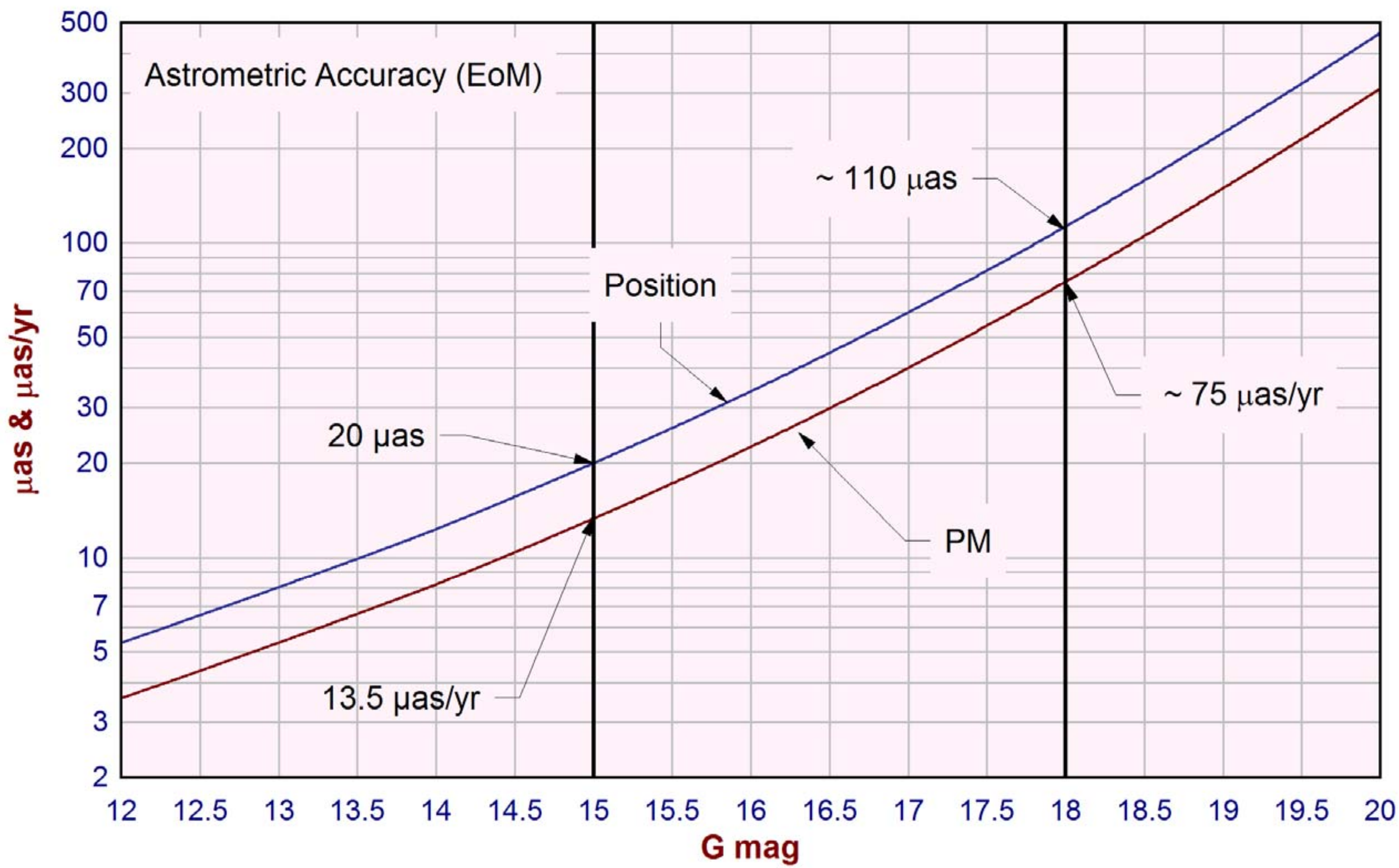
# nom_source	ra	dec	flag_cross	u	b	v	g	r	i
LQAC_000-032_001	0.084999781	-32.350342643	AB------I-KLM	0.00	18.57	17.00	0.00	17.99	17.86
LQAC_000+040_001	0.221173199	40.900498078	AB-D---------	0.00	0.00	0.00	0.00	0.00	0.00
LQAC_017-060_001	17.314480025	-60.830127769	AB-----------	0.00	0.00	0.00	0.00	0.00	0.00

- When > 2 bands available, G magnitude can be estimated
  - otherwise use V or R as substitute
  - or systematic shift if only u,b are available
- Out of 3400 ICRF2 sources, I ended up with 2700+ with an estimated G magnitude

- Magnitude distribution

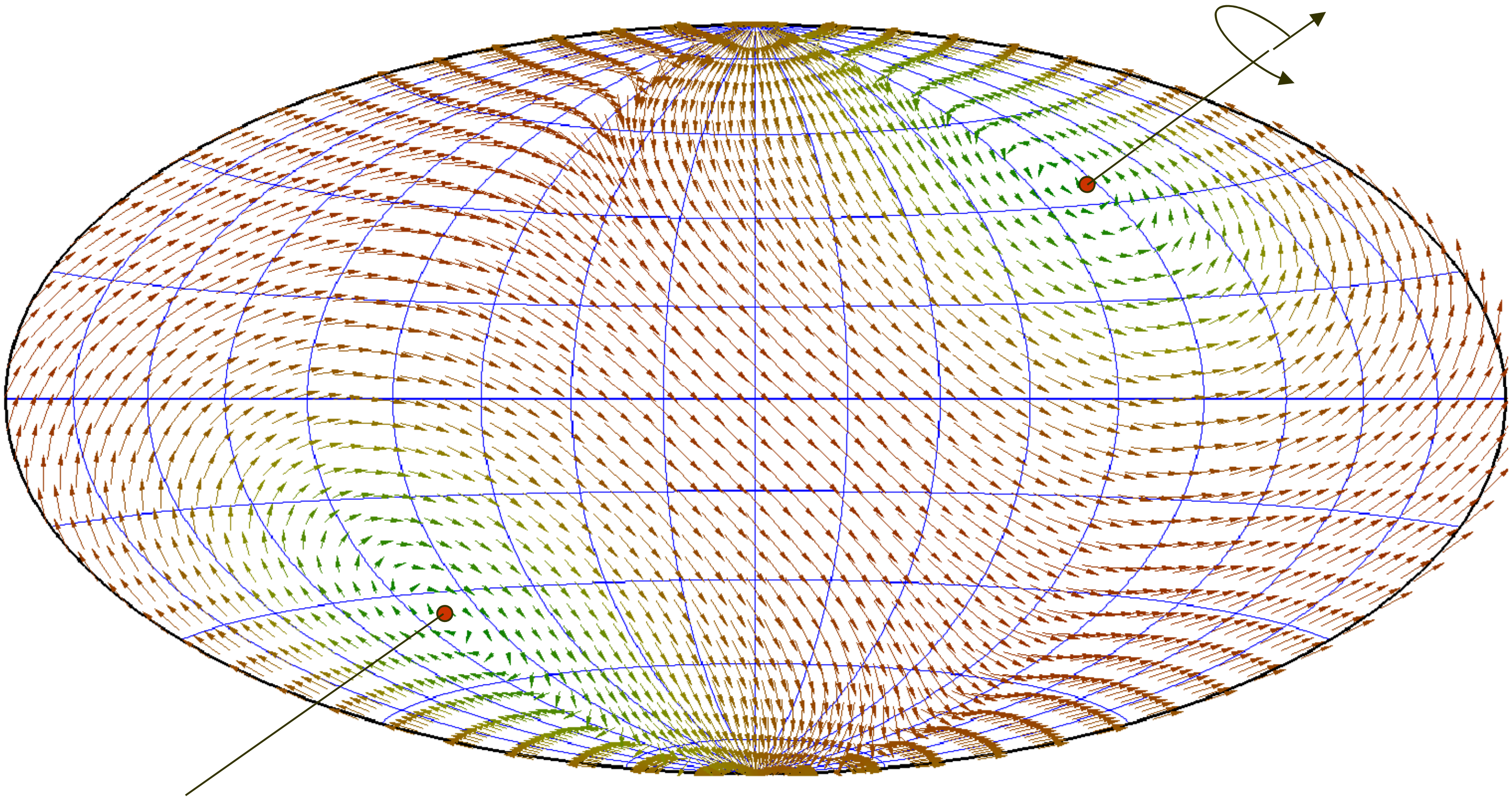


Astrometric Accuracy: QSOs



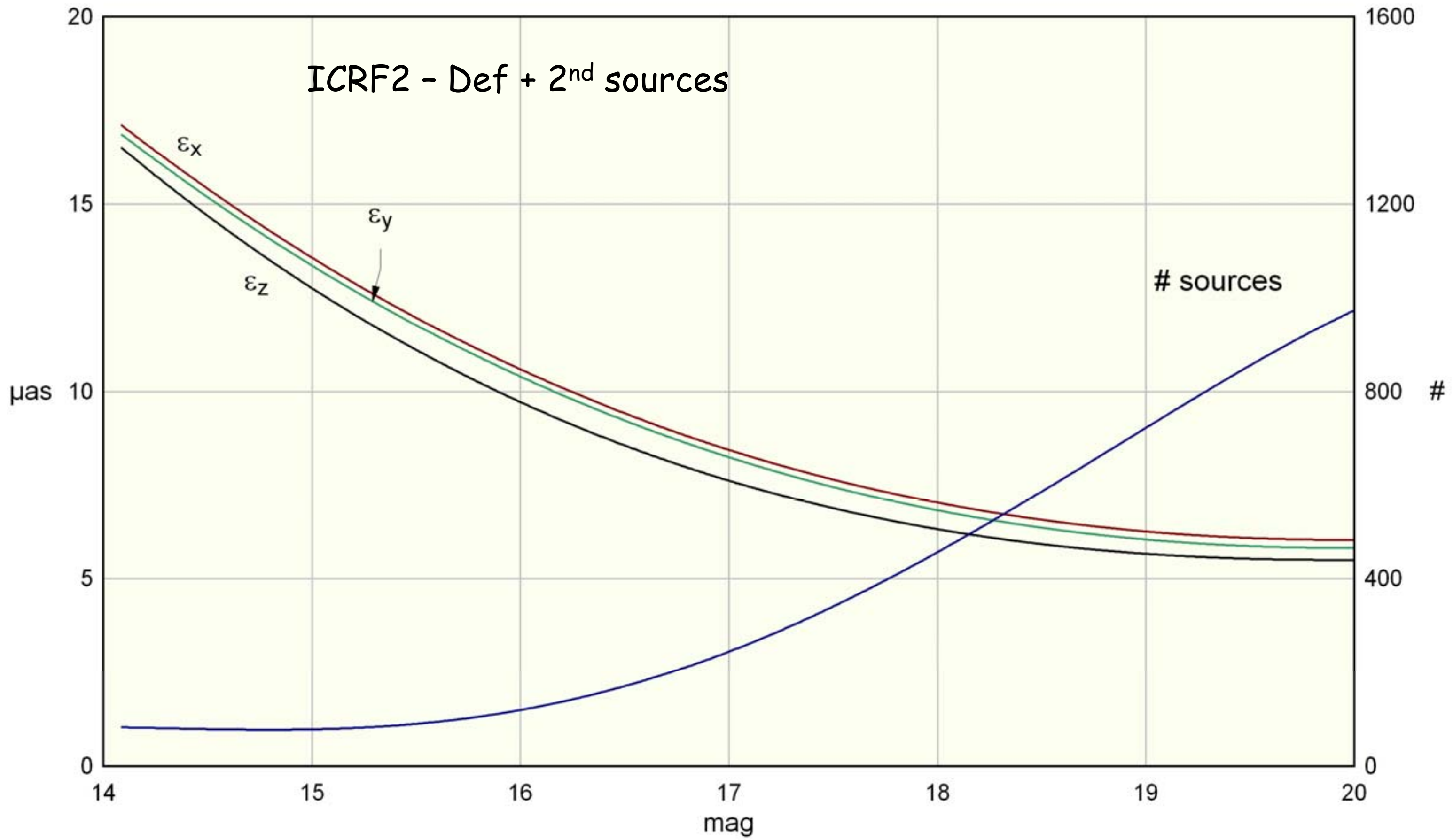
Global Spin and Orientation

- Pattern solved with three parameters (ε_x, ε_y, ε_z)
- same for proper motion with (ω_x, ω_y, ω_z)



Performance in the alignment

- based on the covariance matrix of the rotation model

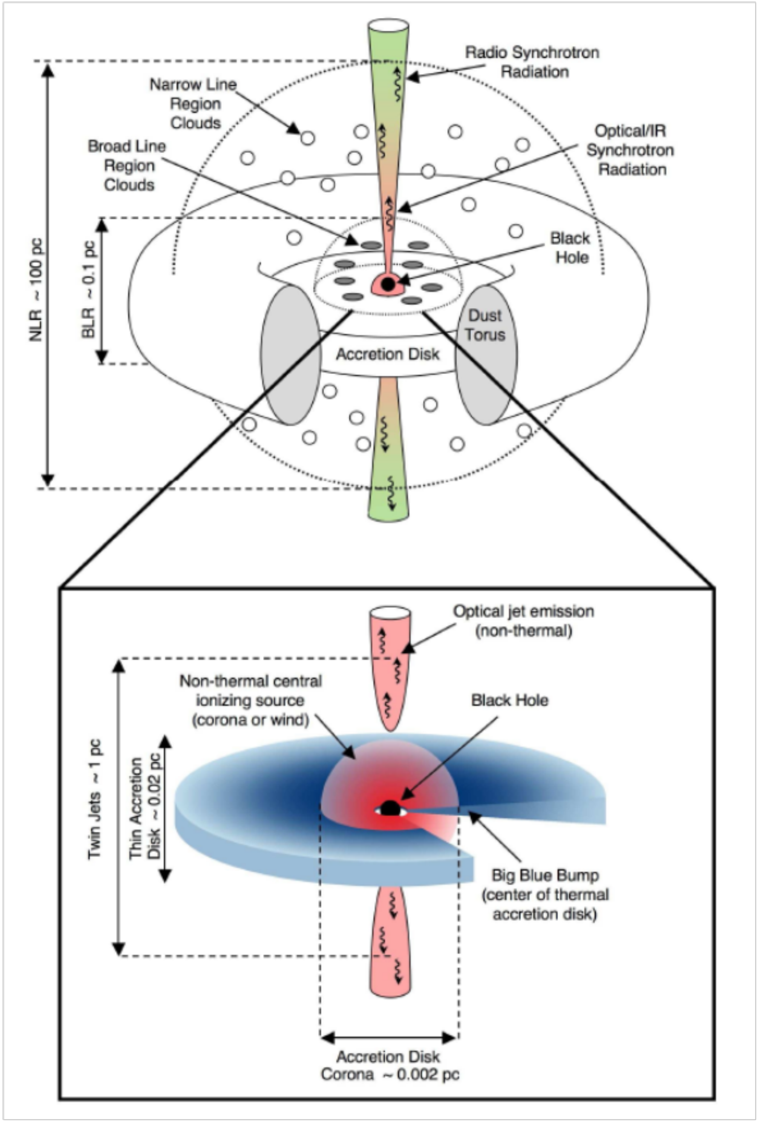


More sources ?

- The number of suitable sources for the alignment may be small
  - stable, no structure, bright enough in the visible
  - About 200 ICRF2 sources are suitable(Bourda et al., 2012)
- A program of selection and observation of additional sources is under way
  - PI : G. Bourda (Obs. of Bordeaux)
- Initially 450 sources pre-selected
  - Flux > 20 MJy, $\delta > -10°$, $V < 18$
  - 400 found detectable in VLBI
- Imaging of 250 sources
  - 120 found without structure
- Astrometric observations of these sources in progress
- We may have > 120 new and high-quality source for the alignment
- ICRF3 will be also an important data set to explore

Radio-Optic offset

- No observational evidence
  - offset < 0.1 to 1 mas
- Emission model can provide insights
- For a distance of 0.1 pc between photocenter and radiocenter ➔ angular distance > 10 μas (z > 1, RG involved)
  - becomes relevant for the alignment on radio ICRF
  - the offsets vectors should be randomly oriented ➔ act as an additional noise



Radio-Optical core-shift

Frequencies in VLBI:

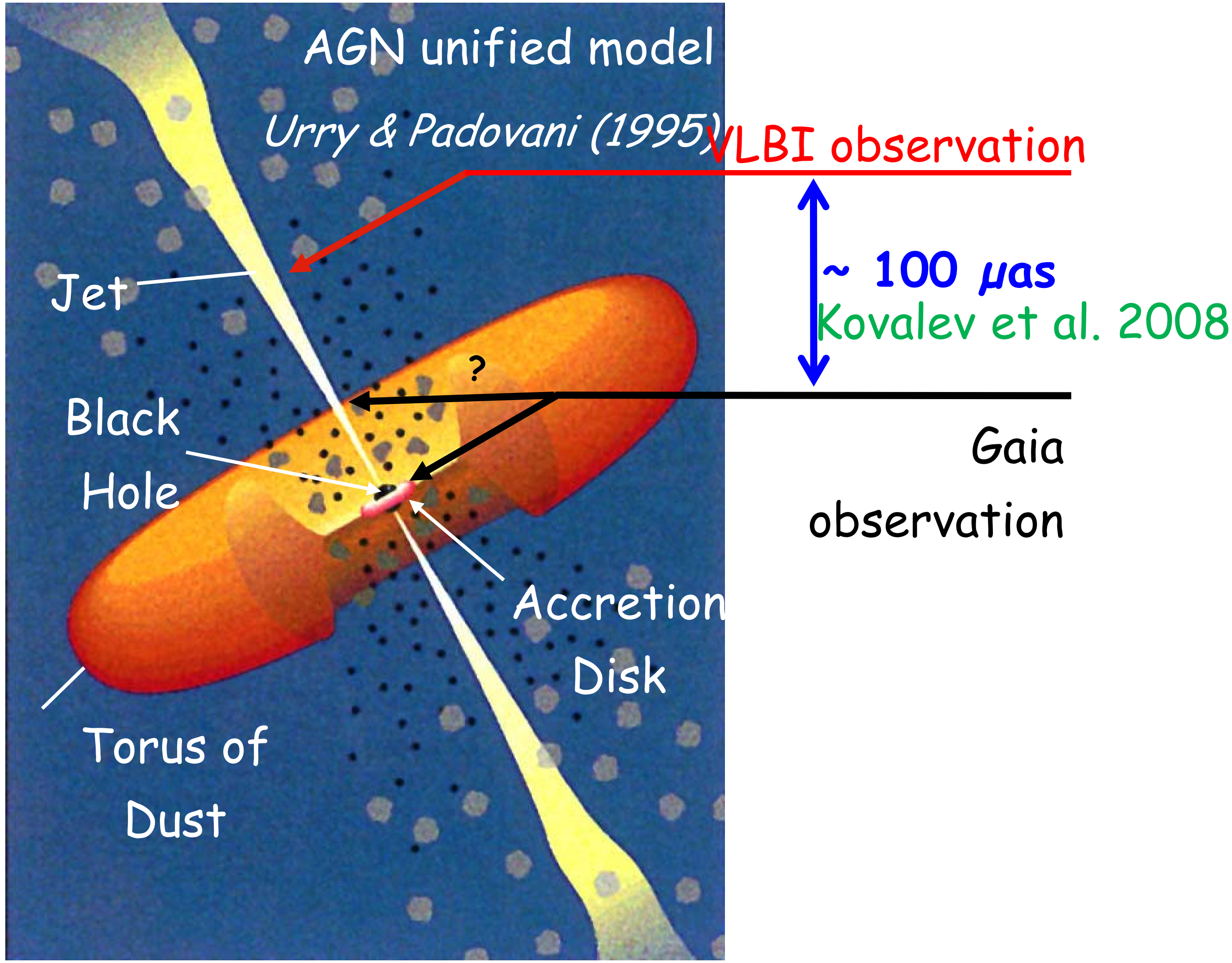
S ~ 2 GHz

X ~ 8 GHz

K ~ 24 GHz

Ka ~ 32 GHz

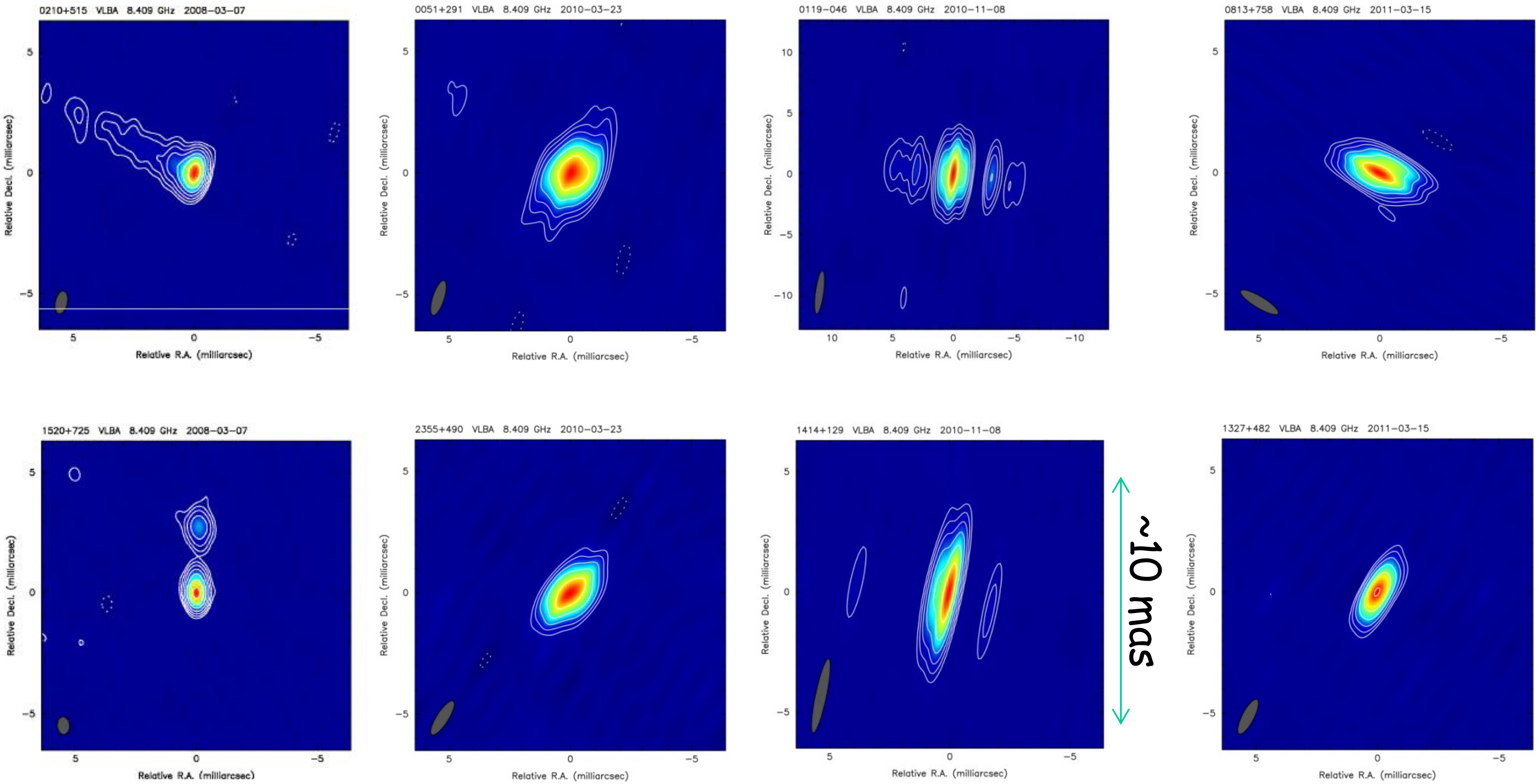
Q ~ 43 GHz



credit : G. Bourda, Obs. Bordeaux

VLBI maps of 'bad sources'

GC030 GC034A GC034BC GC034EF

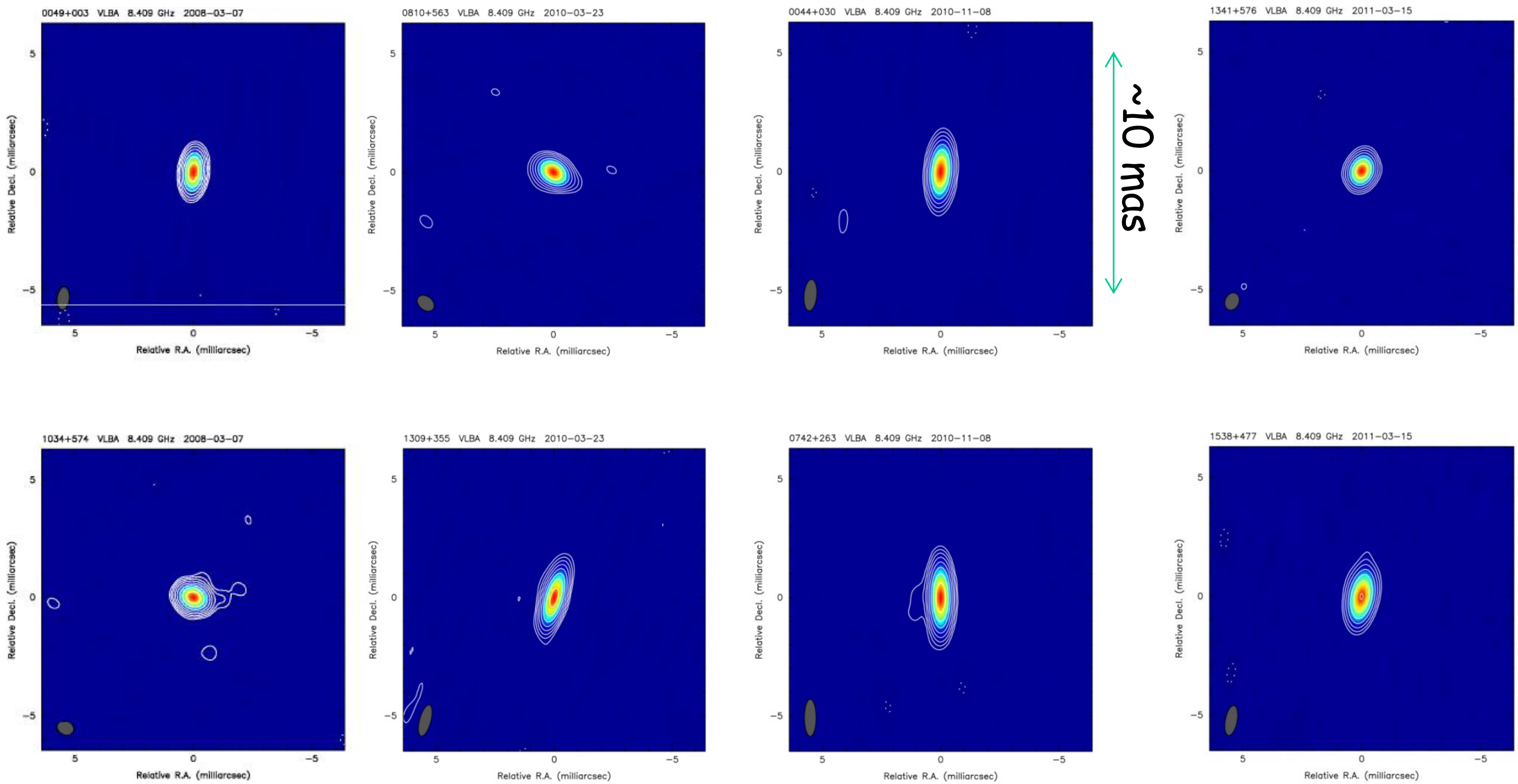


X-band –1st contour level @ 1 – 4%

credit : G. Bourda, Obs. Bordeaux

VLBI maps of 'good sources'

GC030 | GC034A | GC034BC | GC034EF



X-band –1st contour level @ 1 – 4%

credit : G. Bourda, Obs. Bordeaux

Gaia: Practical implementation

- No split between rotation and cosmic acceleration
- Orientation must be determined at the same time
  - cosmic proper motions to be included in the ICRF positions
  - they are not known, but can be computed
- Needs to carry out the alignment:
  - small subset of sources with positions in ICRS
    - ICRF sources and other observed in VLBI ➔ alignment
  - larger subset of EGSs with statistically zero proper motions
    - not necessarily observed with VLBI ➔ rotation

Gaia: Practical implementation

- There are 9 reference frame parameters
  - 3 for the orientation ε_x, ε_y, ε_z
  - 3 for the rotation ω_x, ω_y, ω_z
  - 3 for the cosmic acceleration a_x, a_y, a_z
- They are all determined within the astrometric global solution
  - attitude, position, proper motion will be referred to this frame
  - hopefully the ephemeris is given in a frame nearly identical
    - however the coupling is weak, and an error of 10 mas is acceptable
- This should be carried out in post-astrometric processing