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Fundamental stellar parameters for nearby visual binary stars: η Cas, ξ Boo, 70 Oph and 85 Peg

Helium abundance, age and mixing length parameter for low mass stars

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Abstract. The calibration of nearby visual binary stars on the HR diagram is used to determine the helium abundance, age and mixing length parameter for stars other than the Sun.

Four Population I low mass systems with high-quality data are analysed by means of standard evolutionary stellar models: η Cas, 70 Oph, ξ Boo and 85 Peg.

Complementing these results with those for the Sun and α Cen it is shown that in the framework of the mixing-length formalism to describe convection, a unique value of the mixing-length parameter, equal to the solar one can be used to model these objects.

Except for the 85 Pegasi system, which cannot be explained by means of standard stellar evolutionary models, the helium abundance is determined with a precision of 0.02 (p.e.) and the age with a precision of 2 Gyr (p.e.). A concomitant positive relation between metallicity and helium abundance is found for these stars, corresponding to a mean value of about $\Delta Y/\Delta Z$ $\approx 3\pm2$ (relative helium-to-heavier-elements enrichment parameter) but there is no clear correlation between age and metallicity. The consequences of the results of the use of the new parallaxes from the *HIPPARCOS* mission are briefly discussed.

Key words: solar neighbourhood – Hertzsprung-Russel (HR) and C-M diagrams – stars: fundamental parameters – stars: evolution – binaries: visual

1. Introduction

With the exception of the Sun, modelling a single star is an indeterminate problem because the number of unknown parameters is larger than the observed ones. For a small number of binaries, astrometric and spectroscopic observations are available, which provide stronger constraints on those stellar models.

If the orbit is known and if the system is close enough, high quality measurements of the parallax provide accurate values for luminosities and masses. Spectroscopic observations allow the determination of metallicity and effective temperature, but generally only for the primary component. The effective temperature for the secondary has to rely on the colour index. Both stars are assumed to have a common formation, which implies the same age and the same chemical composition. Therefore, four unknown parameters remain: age, helium abundance and a mixing length parameter for each star. Theoretical stellar models can then be calculated for each component of the binary system. These four unknowns are adjusted to give the best fit between the evolutionary tracks and the observed luminosities and effective temperature for each star. The method has already been applied to α Cen (Noels et al. 1991; Edmonds et al. 1992; Fernandes & Neuforge 1995), and it can be applied to several additional systems to obtain stronger modelling tests.

The knowledge of both helium abundance and age is very important for understanding the galactic chemical evolution. However, for low mass stars the helium abundance Y, in mass fraction, cannot be determined by spectroscopy. This is the case for the Sun where Y is obtained by calibration, using the constraint that the solar model must yield the solar luminosity at the solar age. Solar models using the Livermore radiative opacities (Iglesias et al. 1992) lead to Y $\approx$ 0.28 (Berthomieu et al. 1993; Charbonnel & Lebreton 1993; Dar & Shaviv 1996). This value represents the initial Y, *i.e.* the abundance of the primordial cloud from which the Sun was formed.

The stellar age is very difficult to determine. If the star belongs to a cluster, the age can be estimated by means of isochrones. Empirical methods like the correlation between age and stellar rotation, age and stellar activity or age and stellar kinematics of the Galaxy give only qualitative estimates (Poveda et al. 1994).

The low mass stars have an external convective region with a super-adiabatic layer. The stellar surface is particularly sensitive to the modelling of the convective flux in this layer. When the mixing length theory is used to describe the convection, the stellar model and in particular the super-adiabatic layer remain dependent on a free parameter, α_{MLT}, the mixing length parameter. In the case of the Sun, α_{MLT} is adjusted to reproduce the observed radius of the solar age. The value of α_{MLT} is particularly

Send offprint requests to: J. Fernandes dependent on the treatment of the solar atmosphere, and in particular on the low temperature opacities (Sackman et al. 1990). Therefore, for a certain atmosphere model under a fixed physical hypothesis, the solar α_{MLT} can be determined with an accuracy as good as 10% (Neuforge 1995). Moreover, recent results of convective hydrodynamic simulations seem to indicate that, at least in the main sequence, the α_{MLT} value should be constant during the solar evolution (Ludwig et al. 1995).

The solar α_{MLT} is currently used to model other stars. But one can ask why the value determined for the present Sun should apply to other stars with different masses, ages and/or chemical compositions.

Visual binaries provide useful constraints to answer this question. In recent years there has been renewed interest in the study of $\alpha\ Cen$, the nearest binary. Noels et al. (1991) analysed $\alpha\ Cen$ in the HR diagram and assumed a common origin and the same α_{MLT} value for $\alpha\ Cen$ A and B. This was justified as follows: "As these $\alpha\ Cen$ A and B masses are very close (...) the efficiency of convection should be the same (...)". Consequently, age, metallicity Z, Y, and α_{MLT}, were adjusted. They found the following solution: age = 5 Gyr, Z = 0.04; Y = 0.32 and α_{MLT} = 1.6. The results of Noels et al. (1991) with respect to the age and the helium abundance were confirmed later by other authors (Edmonds et al. 1992; Lydon et al. 1993): $\alpha\ Cen$ is slightly older than the Sun and the helium abundance is higher than the solar one, corresponding to a metallicity also higher than the solar one.

Another interesting result from Noels et al. (1991) concerns α_{MLT}, which was found to be equal to the solar value. This point was confirmed later by Neuforge (1993), using new interior and molecular opacity data in the stellar models and new detailed spectroscopic observations for the effective temperatures of $\alpha\ Cen$ made by Chmielewski et al. (1992).

Edmonds et al. (1992) performed $\alpha\ Cen$ calibrations but did not adopt the hypothesis of a unique α_{MLT}. In contrast to Noels et al. (1991), the Z-value was fixed at 0.026 as a result of a preceding detailed analysis (Furenlid & Meylan 1990). Slightly different α_{MLT} values were claimed for the two components, i.e. $\alpha_{MLT\,A}$ =1.06 and $\alpha_{MLT\,B}$ =1.25 (see also Lydon et al. 1993).

Recently in order to explain the discrepancies between the results of Noels et al. (1991) and Edmonds et al. (1992), Fernandes & Neuforge (1995) pointed out that the determination of the α_{MLT} value for both components was very dependent on metallicity. They performed $\alpha\ Cen$ calibrations for different values of Z, covering the range of the published observed values. The free parameters were Y, age and a different α_{MLT} value for each star. They found that only high Z values (Z $\approx$ 0.038) led to $\alpha_{MLT\,A}$ = $\alpha_{MLT\,B}$, which was equal to the solar value.

Therefore, the knowledge of the correct $\alpha\ Cen$ metallicity is crucial in order to decide if the α_{MLT} solar value can be used for $\alpha\ Cen$ A and B. Very recently Neuforge & Magain (1996) made a new detailed spectroscopic analysis of $\alpha\ Cen$ and they found a metallicity value consistent with the observations of Chmielewski et al. (1992). This indicated that the metallicity of $\alpha\ Cen$ was clearly higher than the solar one, and therefore favoured a "unique value of α_{MLT} for the two components".

The calibration method from Noels et al. (1991) is valid only for solar like objects, where the physics of the models is considered as known except for the treatment of the superadiabatic layer.

When binaries have a more massive component a new parameter appears: the amount of the overshooting from the convective core. The solution is then less severely constrained. This is the case for the $\zeta\ Herculis$ visual binary (Lebreton et al. 1993; Chmielewski et al. 1995) and the spectroscopic binary Al Phe (Andersen 1991).

In this paper we examine the four low mass Population I nearby binary systems η Cas, ξ Boo, 70 Oph and 85 Peg which have the most precise determinations of the observational data.

The paper is organised as follows: in Sect. 2 we present the observational sample studied here, in Sect. 3 we discuss the stellar modelling procedure. In Sect. 4 we present the results and in Sect. 5 we discuss them.

2. The observational sample

2.1. Criteria for the choice of the visual binaries

Starting from Popper's list (1980) we chose the best candidates with respect to the following observational or theoretical criteria:

1. *Independent evolution*: The calibration method could be applied only if the evolution of each component was independent. This could be obtained if both stars were sufficiently separated (typically more than 10 A.U.);
2. *Mass range*: We kept only binaries for which both components had masses in the range $0.6 \leq M/M_{\odot} \leq 1.0$. Stars with masses greater than about $1.1 M_{\odot}$ have a permanent convective core, introducing an additional parameter, the amount of overshooting.
   Moreover in massive stars the microscopic diffusion of chemical elements can be very important and we did not include it in our stellar models (see Sect. 3.1).
   We fixed the lower boundary at $0.6 M_{\odot}$, thereby avoiding difficulties in the treatment of the equation of state (see Sect. 3.1) and the atmosphere;
3. *The best orbits*: We considered the binaries for which the orbits were sufficiently accurate to allow a good determination of the stellar mass. Using the orbit quality scale of Worley & Heintz (1983), of 1 (definite) to 5 (indeterminate) we retained only those classified between 1 and 3, i.e. at least half of the orbit defined or better (see Table 1)
4. *Accurate parallaxes*: We kept the systems for which the parallax was know with an error lower than 4%. Therefore, we expected to have a very precise determination of mass sums ($\leq$10%) and luminosities ($\leq$10%). With the results from the *HIPPARCOS* we expected to improve the parallax error to less than 1% (see Sect. 4.5)
5. *Spectroscopic analysis*: We considered only binaries for which a detailed spectroscopic analysis was performed at

Table 1. Observational data for the selected systhems

Observable	η Cas A	η Cas B	ξ Boo A	ξ Boo B	70 Oph A	70 Oph B	85 Peg A	85 Peg B
Spectral type	G3 V	K7 V	G8 V	K4 V	K0 V	K5 V	G3 V	K6 V
m_v	3.45	7.51	4.70	6.97	4.21	6.00	5.81	9.0
(R-I)		0.59		0.44		0.45		–
$T_{\rm eff}$ (K)	6087±60		5551±20		5322±20		5391±22	
metallicity $[Fe/H]$	-0.31±0.05		-0.21±0.08		0.0±0.1		-0.65±0.10	
Parallax, π(")	0.1684±0.0031		0.1491±0.0036		0.1969±0.0051		0.0796±0.0032	
Period, P (yr)	480±10		151.56±0.17		88.30±0.04		26.27±0.19	
semi-major axis, a(")	11.99±0.02		4.922±0.01		4.560±0.01		0.83±0.02	
Fractional mass ratio, B	0.397±0.01		0.452±0.003		0.445±0.004		0.445±0.008	
Quality of the orbit	3		1		1		1	

least for the primary component, giving a precise measure of its effective temperature and metallicity;

6. *Photometric measurements for the secondary star*: Usually no detailed spectroscopic analysis is available for the secondary, so we considered the binaries for which photometric measurements existed for the secondary. We estimated the effective temperature for the secondary using the colour index (R-I), except for 85 Peg B for which it had not been measured at the time.

Four systems, η Cas, ξ Boo, 70 Oph and 85 Peg, satisfied all those criteria.

2.2. Available observational data

We come now to a brief description of the main characteristics of each binary. Observational data are given in Tables 1 and 2. The errors are absolute errors unless otherwise indicated.

2.2.1. η Cassiopeiae

η Cas (HD 4614) is a nearby visual binary at a distance of $\approx$ 6 pc in the northern hemisphere. It was discovered by Herschel and has been well observed since 1830. In spite of the long binary period ($\approx$ 480 yr) there exists abundant photographic material and it has been possible to determine the orbital elements accurately. Van de Kamp & Worth (1971) computed the orbit and the mass ratio of η Cas taking 251 nights over the period 1912 to 1970. We adopted their orbit for this work.

Van de Kamp (1954) indicates that η Cas A itself is suspected of being an unresolved astrometric binary, with an unseen companion with very small mass, pointing out that η Cas A is found to be over-luminous with respect to the mass-luminosity relation. Until now no evidence has been found for the existence of an unknown companion of η Cas A. This might indicate that the observed over-luminosity is due to η Cas A having evolved with respect to the zero age main-sequence, ZAMS. Indeed, this seems to be the case (see Fig. 1).

Spectroscopic analyses for η Cas A have been carried out by different authors using different methods. We adopted the value published by Gray (1994) based on the analysis of the line-depth ratio between Vanadium and Fe lines, a very sensitive effective temperature indicator for G and K stars (Gray et al. 1996). Gray's determination is in close agreement with the recent determination by Blackwell & Lynas-Gray (1994) who used the infrared flux method, IRFM, yielding 6044 $\pm$ 120 K. The metallicity is taken from Edvardsson et al. (1993).

η Cas is a very interesting candidate for a calibration procedure. First η Cas has a metallicity 3 to 4 times lower than that of α Cen and the comparison of the results for each binary might give an indication of the effect of metallicity on the modelling method. Second, η Cas A is a good candidate for the detection of solar type p-modes (Pery & Libbrecht 1993).

η Cas A is also known to be a slow rotator (v $\sin\ i <$ 6 km/s). Spectral type, photometry and parallax are taken from Gliese & Jahreiss (1991).

2.2.2. ξ Bootis

ξ Boo (HD 131156) is one of the nearest visual binaries (7 pc) for which the orbit is known with great accuracy. The photographic coverage now extends over $90°$ of the orbital arc. Here we adopted the orbit calculated by Hershey (1977).

The results of different recent spectroscopic analyses (Gray 1994; Ruck & Smith 1995) are in close agreement. In order to get uniform values for effective temperature in this work, we chose the Gray et al. (1994) determination. Moreover Gray et al. (1996) have shown that the effective temperature variations of ξ Boo A between 1984-1993 are less than 12 K, which is within the the effective temperature errors (see Table 1).

ξ Boo A is a very slow rotator, v $\sin\ i = 3$ km/s. Spectral type, photometry and parallax come from Gliese & Jahreiss (1991).

2.2.3. 70 Ophiuchi

The spectroscopic visual binary system 70 Oph (HD 165341) is one of our nearest neighbours (5 pc) and is among the first discovered binary stars. It was observed first by Herschel in 1779. The angular separation is always greater than 1.5" and it has completed more than 2.5 orbital revolutions since its discovery. Its orbit is very well known. Moreover, it is one of the unusual cases for which the orbital elements derived from astrometry and

Table 2. Global parameters of the select systems and their uncertainties

	η *Cas* A	η *Cas* B	ξ *Boo* A	ξ *Boo* B	70 *Oph* A	70 *Oph* B	85 *Peg* A	85 *Peg* B
Log$(L/L_\odot)$	0.11±0.03	-1.19±0.08	-0.26±0.03	-1.01±0.08	-0.29±0.03	-0.87±0.08	-0.14±0.04	-1.1±0.1
$T_{\rm eff}$ (K)	6087±60	4036±150	5551±20	4350±150	5322±20	4350±150	5391±22	3900±200
Z	0.009±0.002		0.012±0.002		0.019±0.002		0.004±0.002	
M/$M_\odot$	0.95±0.08	0.62±0.06	0.86±0.07	0.70±0.05	0.89±0.04	0.71±0.04	0.91±0.11	0.73±0.13

from spectroscopy are in very close agreement (Heintz 1988; Batten & Fletcher 1991). We adopted the astrometric orbit of Heintz (1988). The presence of a third short-period object in the system has sometimes been suspected (Heintz 1988), but no perturbation on the radial velocity of 70 *Oph* A has been detected, so that, even if it exists, it would be very small and with no influence on the parameters determination.

The effective temperature was determined by Gray & Johnson (1991) using the line-depth ratio.

The accuracy of the metallicity is quite poor as all the measurements were performed before 1978 with photographic plates. The two most recent determinations yielded $[Fe/H]$ = -0.05 (Peterson 1978) and $[Fe/H]$ = 0.00 (Perrin et al. 1975). We chose the solar metallicity as a representative value of the 70 *Oph* metallicity.

Both components are slow rotators, v $\sin\ i < 25$ km/s. Spectral type and photometry come from Gliese & Jahreiss (1991).

We took the trigonometric parallax given by the U. S. Naval Observatory group (Harrington et al. 1993).

Note that the calibration of 70 *Oph* w ill test if a star of solar metallicity also has a solar helium abundance.

2.2.4. 85 Pegasi

85 *Peg* (HD 224930) is a visual and spectroscopic system in our neighbourhood (12 pc). It is one of the most interesting binaries and has been studied for different reasons: cosmological initial helium abundance (Catchpole et al. 1967), presence of an unseen companion (McCarthy 1983), stellar evolution (Perrin et al. 1977; this work), confrontation between astrometric and spectroscopic solutions for the orbit (Lippincott 1981).

Before 1971, astrometric (Hall 1948) and spectroscopic (Underhill 1963) determinations showed an (abnormal) fractional mass higher than 0.5, viz. the mass of 85 *Peg* B was higher than that of 85 *Peg* A. In order to explain the higher mass of the secondary it was assumed that 85 *Peg* B itself was double.

Later, Feierman (1971) pointed out that the determination of the fractional luminosity of the companion, β, measured only from the apparent magnitude difference, Δm, led to an overestimate of the real value when the binary separation was less than 1.5”, due to a blend effect of the primary on the photographic plate. Taking this effect into account, a more reasonable value of the fractional mass is $\approx$ 0.44 and 85 *Peg* A becomes the more massive companion (Feierman 1971; Lippincott 1981). This result was later confirmed by spectroscopic measurements using CORAVEL (Duquennoy & Mayor 1992) and the corresponding individual spectroscopic masses (0.95±0.2, 0.69±0.2) were in very close agreement with the astrometric ones (see Table 2). Moreover and according to the present available observations the hypothetical binarity of 85 Peg A and B themselves cannot be confirmed (Mayor 1996, private communication). Note that very recently Martin & Mignard (1997) redetermined the mass ratio for 85 *Peg* and for other short-period binaries, using the data from the HIPPARCOS mission. They claimed very similar values for the masses of 85 *Peg* A and B which indicates that the problem with mass determination still remains.

Several metal abundance studies have been performed. The catalogue of Cayrel de Strobel et al. (1992) counts 7 determinations. The weighted metallicity average calculated by Taylor (1994) taking into account the 6 last ones is $[Fe/H]$ = -0.65 $\pm$ 0.10 (see also Gray 1994). We took this value as the reference for 85 *Peg* metallicity. Recently Van’t Veer using the Hα line (1996, private communication) obtained a slightly lower value, $[Fe/H]$ = -0.80.

The new determination of Gray (1994) for the effective temperature of 85 *Peg* A was used; it agrees with the value obtained by Van’t Veer (1996).

85 Peg A is a very slow rotator, v $\sin\ i$ =3.1 km/s. Spectral type, photometry and parallax are taken from Gliese & Jahreiss (1991).

2.3. Observational HR diagram

2.3.1. Effective temperature of the B components

Among the binaries studied here no detailed spectroscopic analysis has been done for any of the B components.

The effective temperatures of the cool companion stars can be determined using the calibrations of the colour index versus effective temperature. Bessel (1994) established a relationship between $T_{\rm eff}$ and $(R-I)_J$ (in which J stands for the Johnson system) coming from a detailed model atmosphere for cool dwarfs. Alonso et al. (1996) argued that $(R-I)_J$ was a very good temperature indicator below 5000 K. The relationship $[(R-I),\ T_{\rm eff}]$ is not strongly dependent on metallicity: for a fixed (R-I), the error on $T_{\rm eff}$ is less than 100 K. As for our sample, only $(R-I)_K$ (K for Kron system) was available. It was converted into $(R-I)_J$ through the relationship from Eggen (1971). Therefore, we considered a resulting $T_{\rm eff}$ error of about 150 K.

The effective temperature for 85 Peg B, for which the $(R-I)$ colour index was not available, was estimated using a blackbody approximation based on the predicted infrared brightness (Johnson & Wright 1983).

2.3.2. Luminosities

In order to derive the stellar luminosities we used the parallaxes and the apparent visual magnitude of Gliese & Jahreiss (1991) and the bolometric correction from Schmidt-Kaler (1981 and references) based on empirical data, for which $M_{bol\odot}$= 4.64. According to Schmidt-Kaler, for Population I stars the relationship of $T_{\rm eff}$ versus bolometric correction is marginally affected by the metallicity. We adopted bolometric correction errors of 0.06 and 0.20 for the first and second component respectively.

The error in luminosity was calculated taking into account the error in bolometric correction and the error in parallax. We adopted typically $\pm$0.03 dex and $\pm$0.08 dex on $\mathrm{Log}(L/L_{\odot})$ for the primary and secondary binary components respectively. 60% and 30% of the total error in the luminosity for components A and B respectively is due to the error in the parallax. In the *HIPPARCOS* data the error in luminosity is mainly dominated by the error in bolometric correction (Baglin 1997).

2.3.3. Metallicity

The mass fraction of heavy elements, Z, was derived assuming $\mathrm{Log}(Z/Z_{\odot}) \approx [Fe/H]$ and $Z_{\odot}$ = 0.019 (Grevesse 1991), for the solar mixture. This relationship is valid for Population I stars which do not present the α-elements enrichment seen in metal deficient stars (Wheeler et al. 1989).

2.3.4. Individual masses

The total binary mass was derived from Kepler's law. We assumed that the error in the sum is due only to the error in parallax (Wielen 1962). Individual stellar masses were then determined using the knowledge of the fractional mass ratio, B.

3. Stellar modelling and calibration

3.1. Input physics for the stellar models

The stellar models were calculated using the "Code d'evolution stellaire adaptative et modulaire", CESAM (Morel 1997). All the stars considered in this work are slow rotators, v $\sin\ i <$ 30km/s, so the spherical approximation is valid. We used the Livermore radiative opacities (Iglesias et al. 1992) complemented at low temperatures (T$\leq$ 10 000 K) by atomic and molecular opacity tables from Kurucz (1992). The opacities were calculated with the solar mixture of Grevesse (1991) corresponding to a solar metallicity Z=0.0190. Convection was described with the classical mixing length theory (Böhm-Vitense 1958). The atmosphere was represented by an Eddington T(τ,$T_{\rm eff}$) law, where τ is the mean optical depth. The nuclear reaction rates were from Caughlan & Fowler (1988).

The equation of state was described by Eggleton et al. (1973), hereafter EFF. This simple analytical formalism was sufficient for our purpose. In fact, Lebreton & Däppen (1988) examined the effect of different equations of state on the position of the ZAMS in the H-R diagram and found that the position of the ZAMS calculated with EFF and with a more realistic equation of state such as MHD (Mihalas et al. 1988) was approximately the same for masses between 0.7$M_{\odot}$ and 1.0$M_{\odot}$. Moreover, for M = 0.6$M_{\odot}$ the difference between stellar models computed with EFF and MHD was inside the typical $T_{\rm eff}$ observational errors for cool stars. These results were in agreement with those obtained using the equation of state designed for low-mass stars and giant planets from Saumon & Chabrier (1992).

The microscopic diffusion of chemical elements changed the stellar structure and was able to affect the position of the stellar model in the HR diagram: in external layers helium diffusion increased the opacity (it increased the H^- contribution) and decreased the effective temperature; in the core, helium increased and produced an increase in the luminosity. Also the heavy elements diffusion mainly changed the opacity in the whole star.

Nevertheless, for low mass stars, the "high density" in external layers broke the diffusion and in the core the diffusion time scale was of the same order as the evolution time scale. Therefore, one could guess that the difference in models for low mass stars with or without diffusion would not be large. This had already been quantified by Edmonds et al. (1992) who performed calibrations of $\alpha\,Cen$ using models with and without helium diffusion. It was found that diffusion did not affect the HR diagram position for $\alpha\ Cen$ B ($\approx$ 0.9$M_{\odot}$) at all and that the difference between the model with helium diffusion and the model without was less than 60 K in the case of $\alpha\ Cen$ A ($\approx$ 1.1$M_{\odot}$). This value was of the same order as the size of the present observational errors in effective temperature for $\alpha\ Cen$ A. We therefore chose not to include the microscopic diffusion in our models.

With these inputs the solar luminosity and radius were obtained at the solar age (4.6 Gyr, Tilton 1988; Guenther 1989) with an initial helium abundance Y = 0.28, and a mixing length parameter for convection α_{MLT} = 1.7.

We calculated evolutionary stellar models for masses between 0.5 and 1.1 $M_{\odot}$, metallicity between 0.004 and 0.019, helium abundance between 0.25 and 0.28 and α_{MLT} between 0.7 and 2.7. Some particular models were evolved from the ZAMS to 6 Gyr.

3.2. Method of calibration

Calibration was inspired by the method developed by Noels et al. (1991) for $\alpha\,Cen$ system. In the case of $\alpha\,Cen$, both stars were supposed to have the same Z, Y, age and α_{MLT}, and were calibrated in the HR diagram with the constraint on observed effective temperature and luminosity.

In our case the calibration method was slightly different because the metallicity was known with sufficient accuracy. Z was therefore an observable of the system which gave a rather strong calibration constraint. We also assumed that both stars had the same age and chemical composition but we allowed different α_{MLT} values.

The sum of the masses was more precise than the derived individual stellar masses, so we fixed MA+MB. We also fixed the metal content.

We searched for the solution which satisfied the constraint of the luminosity and effective temperature for the two stars and which corresponded to the observed metallicity and sum of the masses MA+MB. This yielded the unknowns of the systems: age, Y, $\alpha_{MLT\,A}$ and $\alpha_{MLT\,B}$ and individual masses. The best solution was the one which kept MA and MB inside the observational error bars.

4. Results

The results are presented in Figs. 1, 2, 3, and 4 in which we also indicate the binary position and error box (p.e., probable error).

Table 3 gives the values of MA, MB, Y, $\alpha_{MLT\,A}$, $\alpha_{MLT\,B}$ and age which yielded the observed effective temperature, luminosity and total mass for each binary.

4.1. η *Cas*

The results indicate that η *Cas* is helium-deficient relative to the Sun and slightly younger than the Sun. This is in agreement with the age indications from the chromospheric activity using the Ca II emission line (Hale 1994, Poveda et al. 1994).

Solutions were found adopting the solar α_{MLT} for both stars. However, a slight disagreement between derived and observed individual masses remains. A better mass consistency could be achieved if B $\approx$ 0.36. This value is not completely unrealistic: D. Popper (1980) gives B = 0.38 $\pm$ 0.01 (an average weighted value taking into account the three last publications).

4.2. ξ *Boo*

The calibration of ξ *Boo* gives results very similar to those obtained for η *Cas*, for Y, age and α_{MLT} : ξ *Boo* is younger and helium-deficient relative to the Sun and the same α_{MLT} value was found in ξ *Boo* A as in the Sun.

We suggest that ξ *Boo* is a very young star. This is in agreement with the presence of a large quantity of lithium in the spectrum of ξ *Boo* A (Herbig 1965) as well as considerable chromospheric activity (Ruck & Smith 1995).

The position of ξ *Boo B* agrees with the models only marginally. A better agreement could probably be obtained using a more detailed equation of state. Another possible explanation could be related to $T_{\rm eff}$ determination of ξ *Boo B*. Veeder (1974) established an empirical relation between $T_{\rm eff}$ and (R-I) and found 4270 K for ξ *Boo B*. In Fig. 2 we have also plotted the B component position using Veeder's calibration. It can be seen that the differences between the model and the observation are clearly reduced if we use Veeder's instead of Bessel's value. Moreover, our results provide an indirect determination of the gravity of ξ *Boo A*. We find Log g = 4.6.

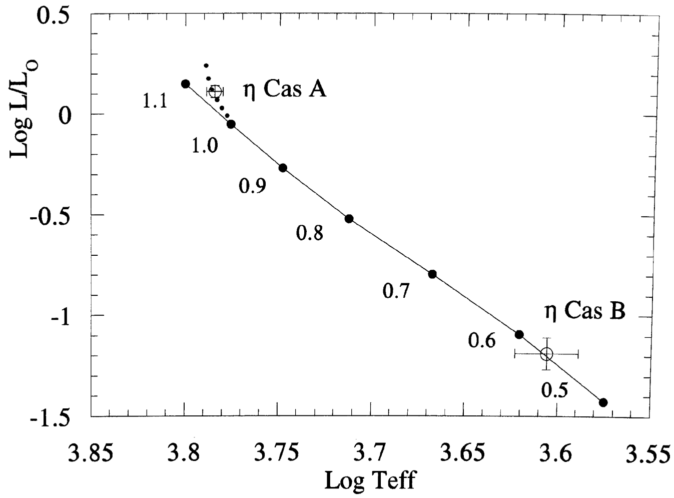

Fig. 1. Calibration of the η *Cas* system in the HR diagram ZAMS models corresponds to 0.5$\leq M/M_\odot \leq$1.1, Z=0.009 and Y=0.25. Small black dots represent the $1.0 M_\odot$ evolutionary track from 0 to 6 Gyr in steps of 1 Gyr.

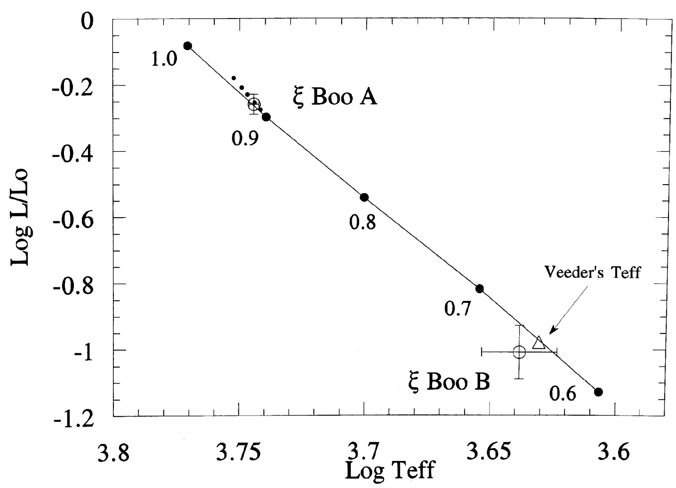

Fig. 2. Calibration of the ξ *Boo* system in the HR diagram. ZAMS models correspond to 0.6$\leq M/M_\odot \leq$1.0, Z=0.012 and Y=0.26. Small black dots represent the $0.9 M_\odot$ evolutionary track from 0 to 6 Gyr in steps of 1 Gyr; the open triangle represents the position of ξ *Boo* B using Veeder's calibration (see text).

As for η *Cas* a change in B (of about 0.42) is needed to get exact agreement between the observed and calibrated mass values.

4.3. 70 Oph

The individual masses found are in very close agreement with the observed values confirming that the metallicity of 70 *Oph* is very close to the solar one. The values of α_{MLT} and Y are the same as for the Sun. The age is lower than the solar age: 3 Gyr is probably an upper limit. This seems to be in agreement with its rotation velocity (v $\sin\ i \approx$ 16 km/s), which is high enough for a main-sequence low-mass star. Note, however, that

Table 3. Results of the calibration and corresponding uncertainties (p.e, probable error)

Observable	η *Cas* A	η *Cas* B	ξ *Boo* A	ξ *Boo* B	70 *Oph* A	70 *Oph* B	85 *Peg* A	85 *Peg* B
$M/M_\odot$	1.00±0.04	0.57±0.07	0.90±0.04	0.66±0.07	0.90±0.04	0.70±0.07	no solution	no solution
Helium, Y	0.25±0.02		0.26±0.02		0.28±0.02		no solution	
α_{MLT}	1.7±0.3	1.7	1.7±0.3	1.7	1.7±0.3	1.7	no solution	no solution
age	4±2		2±2		3±2		no solution	

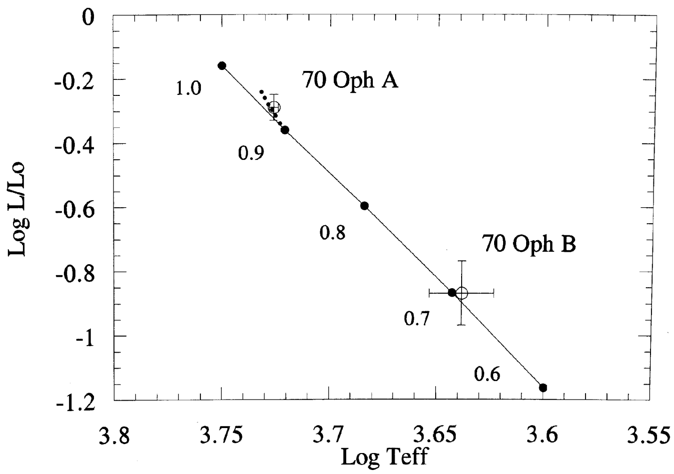

Fig. 3. Calibration of the 70 *Oph* system in the HR diagram. ZAMS models correspond to 0.6$\leq M/M_\odot \leq$1.0, Z=0.019 and Y=0.28. Small black dots represent the 0.6$M_\odot$ evolutionary track from 0 to 6 Gyr in steps of 1 Gyr.

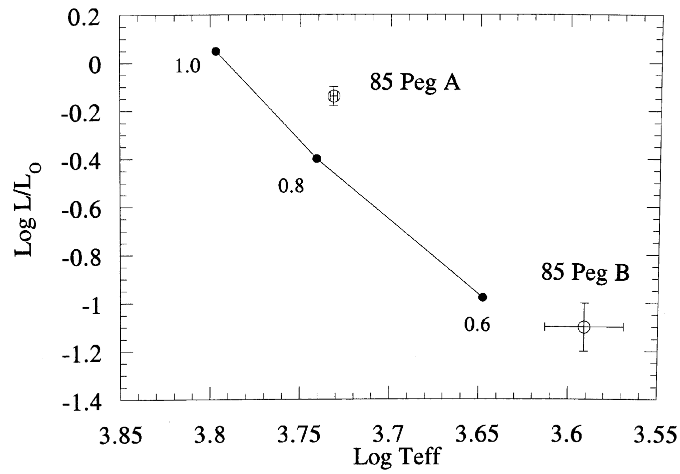

Fig. 4. 85 *Peg* system position in the HR diagram. ZAMS models correspond to 0.6$\leq M/M_\odot \leq$1.0, Z=0.004 and Y=0.23

the dispersion of the age vs. stellar rotation relation is quite high, amounting to more than 2 Gyr (Dorren et al. 1994).

4.4. 85 Peg

Fig. 4 indicates that the calibration by stellar models does not seem possible: 85 *Peg* A and B appear to be too cold and/or over-luminous with respect to the ZAMS. Perrin et al. (1977) have already mentioned this problem. On the other hand taking into account the observed masses (see Table 2) we do not expect significant evolutionary effects in the HR diagram for 85 *Peg* A and B.

It is tempting to play with the stellar parameters in order to obtain a solution. Only models with extremely low helium (Y $<$ 0.20) and high age ($>$20 Gy) could fit the HR diagram position of 85 *Peg* A, which seems definitively unrealistic since the primordial helium is estimated to be Y$\geq$ 0.23$\pm$ 0.01 (Balges et al. 1995; Pagel et al. 1995) and the age of the Universe to be between 10-20 Gy.

A possible alternative solution would be to change α_{MLT}. It is possible to fit the 85 *Peg* A position by decreasing α_{MLT} to about 1.0 (Axer et al. 1995) but a similar change of α_{MLT} will not fit 85 *Peg* B position (the sensibility of a model of 0.6$M_\odot$ to α_{MLT} is not so high). As a consequence the ZAMS slope for 85 Peg would become very strange.

Are the observations correct? The $[Fe/H]$ and effective temperature of 85 Peg A have been determined several times. The good agreement between these determinations gives some confidence to the values obtained.

Perrin et al. (1977) also discuss a similar situation for μ Cassiopeiae A, another metal-poor nearby star, so the possibility of a relationship between the "85 *Peg* problem" and low metal abundance should be checked. The mixture of the heavy elements could differ from the solar one, i.e. oxygen might be overabundant with respect to iron as is the case in Pop II metal poor stars (Axer et al. 1995). However, it would lead to a global shift of the main sequence (Lebreton et al. 1997) and will not help to solve this difficulty.

4.5. Accuracy of the results

How accurate are the inferred parameters? The errors presented below reflect only the errors in the derived parameters taking into account the errors in the stellar effective temperature, luminosity and metallicity (see Sect. 2.3). The error in the observed metallicity was considered as an effective error in $\mathrm{Log}(L/L_\odot)$ and $T_{\rm eff}$ according to ΔZ/ΔLog$(L/L_\odot)$ = - 0.045 and ΔZ/Log $\Delta T_{\rm eff}$ = - 0.2 (coefficients are obtained with the help of theoretical ZAMS models for the range of masses and chemical compositions considered in this work). Considering a typical observational error in the metallicity, ΔZ = 0.002, the effective errors in $\mathrm{Log}(L/L_\odot)$ and $T_{\rm eff}$ were respectively 0.045

Table 4. *HIPPARCOS* versus ground-based parallaxes for the selected systems

Parallax	$\eta\,Cas$	$\xi\,Boo$	70 Oph	85 Peg
this work	0.1684±0.0031	0.1491± 0.0036	0.1969±0.0051	0.0796±0.0032
HIPPARCOS	0.16799±0.00062	0.14926±0.00076	0.19662±0.00138	0.08063±0.00303

and 0.010 for the first component and 0.091 and 0.015 for the second one.

1. *Mass*. The error in the mass of the primary star is mainly due to the evolution (the range of the evolutionary track was considerably greater than the error box).
   The mass error of the secondary was mainly due to the error box size (the evolutionary track always remained in the error box): ΔM/ΔLog $T_{\rm eff} \approx$ 5.5 and ΔM/$\Delta{\rm Log}(L/L_\odot) \approx$ 0.5 corresponded to ΔM $\approx \pm$ 0.04 (p.e.) for the primary star and ΔM $\approx \pm$ 0.07 (p.e.) for the secondary.
2. *Helium abundance*. The error in the helium abundance was determined taking into account the observational errors for the primary star. Theoretical stellar models for 0.8$\leq$ M/$M_\odot$$\leq$1.0 indicated that ΔY/ΔLog $T_{\rm eff} \approx$ 1.5 and ΔY/$\Delta{\rm Log}(L/L_\odot) \approx$ 0.3. These values corresponded to ΔY $\approx \pm$ 0.02 (p.e.).
3. *Mixing length parameter*. The outer layers of the stellar convective region (i.e. the super-adiabatic region) were very sensitive to the parameter α_{MLT}. A change of α_{MLT} implied a change of stellar radius and as a consequence an effective temperature variation (the luminosity remains quasi-unchanged).
   The $1.0 M_\odot$ ZAMS model indicated that a variation from α_{MLT} = 1.7 to 2.7 implied an increase of about 300 K in effective temperature, typically $\Delta\alpha_{MLT}$ /ΔLog $T_{\rm eff} \approx$ 50. However as the mass decreased, the density increased and the convective efficiency also increased, so the super-adiabatic region became thinner. As a consequence the effective temperature became less sensitive to α_{MLT}. For M=0.6$M_\odot$, variations of α_{MLT} = 1.7 to 2.7 led only to an increase of $\approx$ 30 K in effective temperature and for M<0.4 $M_\odot$ the stellar structure became completely independent of α_{MLT} on the main sequence.
   The error in α_{MLT} was determined by taking into account the evolution of the primary component; a typical error of 50 K in the effective temperature of the primary gives $\Delta\alpha_{MLT}$ = $\pm$ 0.3 (p.e.).
4. *Age*. Because the location of low-mass stars models in the HR diagram was very weakly dependent on age, the age of the system depended essentially on the evolution of the primary. For a typical evolutionary model of 1.0 $M_\odot$, Δt/ΔLog $T_{\rm eff} \approx$ 250 and Δt/$\Delta{\rm Log}(L/L_\odot) \approx$ 33. This gave Δt = $\pm$ 2 Gyr (p.e.).
5. *Improvements from the HIPPARCOS data.* Just before this work was finished, we had access to the parallax coming from the *HIPPARCOS* mission, thanks to the proposal INCA011 (Baglin et al. 1982). They are compared to the parallaxes used in this work (Table 4.).
   For $\eta\,Cas$, 70 Oph, $\xi\,Boo$ the differences between the *HIPPARCOS* parallaxes and those used in this work were very small (typically 10 times lower than the ground-based parallax errors). As a consequence, the stellar masses and luminosities remained unchanged. Therefore, no changes of the calibrated parameters, Y, α_{MLT} and t, were expected for these binaries using the *HIPPARCOS* data.
   As for 85 Peg, the difference between the ground-based and *HIPPARCOS* parallaxes was still too small to explain the strange HR diagram position of the binary.
   Nevertheless the use of the *HIPPARCOS* data in this work will be studied more carefully in future work. In fact very recently Söderhjelm et al. (1997) claimed that using the *HIPPARCOS* parallaxes with old orbits could be a bad idea taking into account that *HIPPARCOS* had also made observations of the orbital motion of visual binaries, and that some orbits should be reexamined, in particular those with low periods.

5. Discussion and conclusions

We studied the modelling of four nearby visual binary stars.

The modelling of $\eta\,Cas$, $\xi\,Boo$ and 70 Oph was made by fixing effective temperature and luminosity for both stars as well as the metallicity and the total binary mass.

We did not find an acceptable solution explaining the 85 Peg position in the HR diagram with our stellar modelling procedure. This position was also incompatible with the same mass for both stars.

Within the error bars the individual masses derived through modelling were in agreement with the astrometric values coming from the orbital analysis. For $\eta\,Cas$ and $\xi\,Boo$ a decrease of about 10% in the mass ratios allowed them to fit the observational values precisely. However, this change was higher than the claimed accuracy of the mass ratio of 2%.

The present work shows that a single value of α_{MLT}, viz. the solar value, can be used to model low-mass stars of Population I. This result is independent of mass, age and chemical composition.

1. The uniqueness of α_{MLT} for low-mass stars suppresses an arbitrary free convection parameter. This is particularly important as the convection description has been one of the main sources of theoretical inaccuracy in the modelling of low mass stars.
   Moreover the uniqueness of α_{MLT} reduces the number of unknown parameters in the comparison between theory and observations.
2. For a fixed chemical composition and taking into account that the dependence on α_{MLT} decreases with mass, the

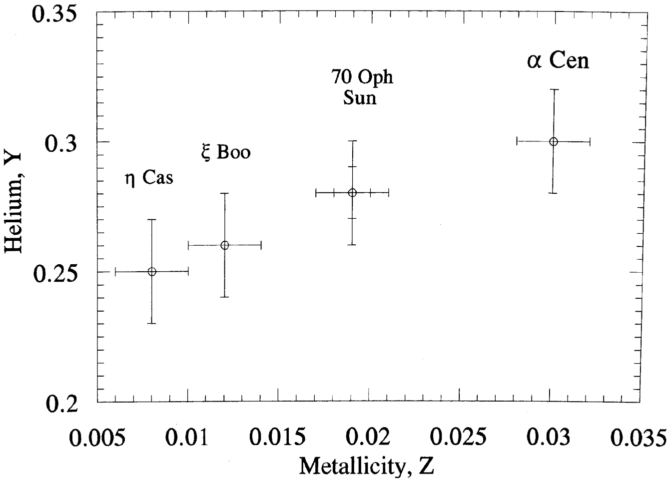

Fig. 5. Helium versus metal abundance: α $Centauri$ (Fernandes & Neuforge 1995) and other stars (this work)

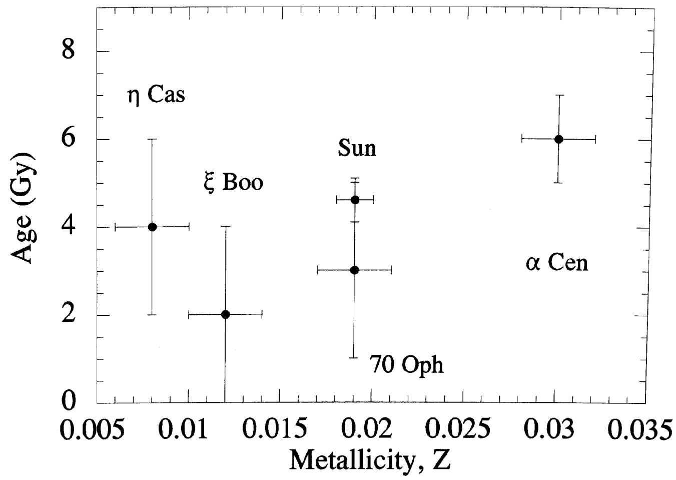

Fig. 6. Age versus metal abundance (same systems as in Fig. 5)

uniqueness of α_{MLT} for low-mass stars fixes the shape (and the slope) of the theoretical ZAMS (Fernandes et al. 1996a), which is then a very well defined curve.

The present work provides precise determinations of age and helium abundance for stars other than the Sun. The accuracy in age is 2 Gyr (p.e.) and the accuracy in the helium abundance is 0.02 (p.e.).

As the chemical composition of 70 Oph is the same as the solar one, 70 Oph A and B and the Sun define the solar mass-luminosity relation between 0.7 and $1.0 M_{\odot}$. A positive correlation between metallicity and helium as shown in Fig. 5, where the Sun and α Cen have been included, should be noted.

If one fits a straight line to the points in Fig. 5, it has a slope of about 3. This slope represents only a mean value of $\Delta Y/\Delta Z$ for this sample and nothing can be concluded about the hypothetical invariance of $\Delta Y/\Delta Z$ in the solar neighbourhood. This mean value is in agreement with recent studies about the spread of the main sequence for low mass stars due to the chemical composition variations discussed by Fernandes et al. (1996b). Using their Eq. (2a), between the enrichment ratio and the main sequence spread, Δ $M_{bol} \approx$ 0.30 is found, in contrast with Perrin et al. (1977) where it was concluded that the main sequence was not enlarged by variations in chemical composition.

An indirect result from our work concerns the age-metallicity relationship. Our results do not support a linear relationship between age and metallicity among Population I stars. A possible explanation is that the stars that we observed in the solar neighbourhood were probably formed in different regions of the Galaxy with different star formation rates and different initial chemical conditions. So there are no reasons to expect a monotonous age-metallicity relation for stars of the solar neighbourhood (see also François & Matteucci 1995).

We have not been able to explain the position in the HR diagram of 85 Peg A and B which confirms the difficulty of modelling low metallicity stars of the solar neighbourhood (Lebreton et al. 1997).

Unfortunately, only a few systems can be studied at this level of accuracy, due to the quality of the observational data, in particular the $[Fe/H]$ ratio, the effective temperature scales and the bolometric correction for the cool components. The new *HIPPARCOS* parallaxes combined with the improved spectroscopic data, with particular attention paid to metallicity, will provide the necessary material to increase both the accuracy of the calibration of these objects and the number of systems to which they can be applied. Detailed tests of the stellar structure modelling, as well as a better knowledge of the chemical history of the solar neighbourhood will then be achievable.

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