THE SYSTEM OF 44 i BOOTIS

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ABSTRACT

Two series of photoelectric observations of the combined light of this triple system are discussed, one series by Stebbins and Huffer in 1930 and the other by the author in 1947. Approximate photometric elements are derived from the 1930 light-curve. A difference in the total light of the system at the two maxima is found from the 1930 observations; and also a peculiar distortion of the light-curve at maximum, suspected in the 1930 results, is confirmed by the 1947 observations.

A combination of the photometric and spectroscopic elements yields masses of 1.0 ☉ and 0.5 ☉ for the eclipsing components. These masses are at variance with the near-equality of luminosity of the stars, inferred from the depths of the minima and the relative intensities of the spectral lines. This apparent violation of the mass-luminosity relation is characteristic of the W Ursae Majoris type systems and is perhaps due to the presence of large amounts of reflected light within the system which permits the secondary component to be seen only by reflection. This hypothesis is supported by the relatively small orbital inclinations derived for most of these systems.

A combination of the spectrographic, visual, and photometric orbits of the triple system shows the brighter visual component, $m = 1.0$ ☉, and $M_v = +4^{\mathrm{m}}.63$, and the brighter eclipsing component, $m = 1.0$ ☉, and $M_v = +4^{\mathrm{m}}.6$, to be very similar to the sun.

It is found that the change in period of the eclipsing system is not entirely due to motion in the visual orbit. An examination of the well-studied W Ursae Majoris type systems indicates that intrinsic changes in the period may be characteristic of these stars.

The visual binary now known as 44 i Bootis was discovered by W. Herschel in 1781. On the discovery night he entered in his observing book:

> Aug. 17 [1781]. Double, considerably unequal. Both W [white]. With 227 [power] they seem almost to touch or at most $\frac{1}{4}$ diameter of S [southernmost star] asunder; with 460, $\frac{1}{2}$ or $\frac{3}{4}$ diameter of S. This is a fine object to try a telescope, and a miniature of alpha Geminorum.[1]

The most recent discussion of the observations of this important double star is the one published by K. Aa. Strand[2] in 1937. His final elements, which we shall adopt throughout the ensuing discussion, are

$$P = 219.5 \text{ years}\,, \qquad a = 3''.609\,,$$
$$T = 1790.0\,, \qquad i = 83^{\circ}.46\,,$$
$$e = 0.42\,, \qquad \omega = 58^{\circ}.85\,.$$

The fainter component of the visual pair was found by members of the Mount Wilson staff to have a spectrum resembling that of W Ursae Majoris,[3] a short-period variable star the spectrum of which is characterized by rotationally broadened lines. J. Schilt,[4] from plates taken with the 60-inch reflector at Mount Wilson in 1926, found the star to be a variable as suspected and with a period of something more than a quarter of a day. It is of some interest to note that the variation of one or both components had been previously suspected by several observers of the visual pair. These early observations are summarized by Miss Agnes Clerke.[5]

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[1] *Phil. Trans. R. Soc.*, **72**, 216, 1790.

[2] *Ann. Leiden*, **18**, 98, 1937.

[3] W. S. Adams *et al.*, *Mt. W. Contr.*, No. 199; *Ap. J.*, **53**, 53, 1921.

[4] *Mt. W. Contr.*, No. 316; *Ap. J.*, **64**, 215, 1926.

[5] *Nature*, **39**, 55, 1888.

In 1929, G. P. Kuiper[6] investigated the photographs of the visual pair taken by Hertzsprung in 1915 and 1919 and by Münch in 1922 and 1926 and confirmed the variability found by Schilt. There have been several investigations of the light-variations of the fainter component of the visual double, including a photographic study by Plaut[7] at Leiden in 1939, a photoelectric study by Shapley and Calder of Harvard in 1937,[8] a two-color study by Nikonov in 1940,[9] and an unpublished photoelectric investigation by Stebbins and Huffer at the Washburn Observatory in 1930. These last observations, which have been turned over to me for discussion, and an additional series of observations by myself in 1947 form the basis of the present investigation.

TABLE 1

COMPARISON STARS FOR 44 i BOOTIS

HR		R.A. (1900)	Decl. (1900)	m_v	Sp.
5618............	44 i Boo	$15^h00^m.5$	+48° 3′	4.86	G2–G2
5581............	....	14 53.0	+50 3	5.68	F5
5635............	....	15 3.4	+54 56	5.21	G2

TABLE 2

CONSTANCY OF THE COMPARISON STARS

JD	5635>5581	Residual	No.	Observer*	JD	5635>5581	Residual	No.	Observer*
2426040...	$0^m.012$	$-0^m.010$	16	S	2426068...	$0^m.026$	$+0^m.004$	16	S
44...	.025	+ .003	12	H	74...	.023	− .001	18	S
47...	.020	− .002	18	S	88...	.027	+ .005	10	S
54...	.020	− .002	12	H	89...	0.025	+0.003	8	S
55...	.021	− .001	18	S					
63...	0.023	+0.001	16	H	Mean.	0.022	±0.003	142	.....

* S = Stebbins; H = Huffer.

The Washburn observations of 1930 consist of measures made of the combined light of the visual pair on 10 nights. The two comparison stars used are described in Table 1. A neutral shade-glass was necessary on the variable to equalize the light. A test of the constancy of the comparison stars is given in Table 2. For some reason, the result on the first night is discordant by 0.01 mag., but the two stars may be considered constant. The normal differences of magnitude, in the sense of HR 5581 being brighter than 44 i Bootis, are given in Table 3, where the phases are computed from the elements

$$\text{Minimum} = \text{JD } 2426058.8554 + 0^d.26780708\,E\,, \qquad (1)$$

and n indicates the number of sets, of three observations each, comprising each normal. These normals are plotted in Figure 1.

There is some doubt as to the difference in magnitude between the components of the visual pair at maximum light of the eclipsing system; but for the present purpose, in order to secure approximate photometric elements, the value 0.76 mag. in photographic

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[6] *B.A.N.*, No. 165, 1929.

[7] *B.A.N.*, No. 3 1, 1939.

[8] *Harvard Bull.*, No. 907, p. 13, 1937.

[9] *Bull. Abastumani Ap. Obs.*, **4**, 1, 1940.

light, given by Schilt, has been adopted. The normal magnitudes, after the observations were reduced to the light of the eclipsing system alone, are given in the third column of Table 3. The light of the uneclipsed stars was found to vary as

$$\begin{array}{l} l = 1.0000 - 0.0383 \sin\theta - 0.0296 \cos\theta - 0.2302 \cos^2\theta\,, \\ \qquad \pm 0.0019 \pm 0.0014 \qquad \pm 0.0024 \qquad \pm 0.0056 \end{array}$$

with the resulting probable error of a single normal between minima of ±0.005 mag. or ±0.002 mag. referred to the total light of the visual components. For purposes of comparison, (1) the mean photoelectric light-curve determined by Shapley and Calder in 1935 and (2) the photographic curve determined by Plaut in 1939 were reduced with the same assumption as to the magnitude difference in the visual pair, 0.76 mag., as was adopted for the reduction of the 1930 Washburn curve.

TABLE 3

NORMAL MAGNITUDES OF 44 i BOOTIS

Phase	Obs. Δm	Corr. Δm	O−C	Phase	Obs. Δm	Corr. Δm	O−C
$0\overset{\mathrm{p}}{.}000$	$-0\overset{\mathrm{m}}{.}050$	$+0\overset{\mathrm{m}}{.}530$	$0\overset{\mathrm{m}}{.}000$	$0\overset{\mathrm{p}}{.}491$	$-0\overset{\mathrm{m}}{.}012$	$+0\overset{\mathrm{m}}{.}376$	$-0\overset{\mathrm{m}}{.}001$
.020	− .043	+ .500	+ .008	.526	− .004	+ .345	+ .003
.040	− .027	+ .436	− .013	.558	+ .018	+ .265	+ .006
.067	− .004	+ .456	+ .005	.590	+ .047	+ .165	− .004
.089	+ .001	+ .326	− .003	.618	+ .068	+ .096	− .009
.108	+ .028	+ .233	.000	.641	+ .078	+ .065	− .004
.130	+ .040	+ .188	.000	.672	+ .095	+ .012	.000
.165	+ .058	+ .128	.000	.697	+ .102	− .009	+ .001
.187	+ .068	+ .096	− .007	.715	+ .110	− .033	+ .003
.211	+ .074	+ .077	+ .004	.743	+ .113	− .043	+ .011
.245	+ .085	+ .042	− .011	.772	+ .113	− .043	+ .008
.277	+ .079	+ .061	− .005	.803	+ .102	− .009	− .002
.304	+ .077	+ .068	+ .018	.831	+ .080	+ .058	− .006
.333	+ .077	+ .068	.000	.871	+ .058	+ .128	+ .005
.361	+ .059	+ .125	+ .009	.898	+ .045	+ .171	+ .007
.396	+ .048	+ .161	+ .009	.920	+ .022	+ .250	+ .003
.420	+ .031	+ .219	− .009	.949	− .004	+ .345	+ .018
.446	+ .016	+ .272	+ .013	0.974	−0.040	+0.488	−0.005
0.466	−0.002	+0.338	−0.004				

$$\begin{array}{l} (1)\ l = 1.0000 - 0.0035 \sin\theta - 0.0227 \cos\theta - 0.2270 \cos^2\theta\,; \\ \qquad \pm 0.0019 \pm 0.0023 \qquad \pm 0.0038 \qquad \pm 0.0096 \end{array}$$

$$\begin{array}{l} (2)\ l = 1.0000 - 0.0200 \cos\theta - 0.2362 \cos^2\theta\,. \\ \qquad \pm 0.0031 \pm 0.0040 \qquad \pm 0.0100 \end{array}$$

It is interesting to note in this connection that an analysis of the light between the minima observed by Schilt,[3] for which the intensity of the photographic images of the eclipsing system alone were measured, gives

$$\begin{array}{l} (3)\ l = 1.0000 - 0.0143 \cos\theta - 0.2145 \cos^2\theta\,. \\ \qquad \pm 0.0087 \pm 0.0092 \qquad \pm 0.0228 \end{array}$$

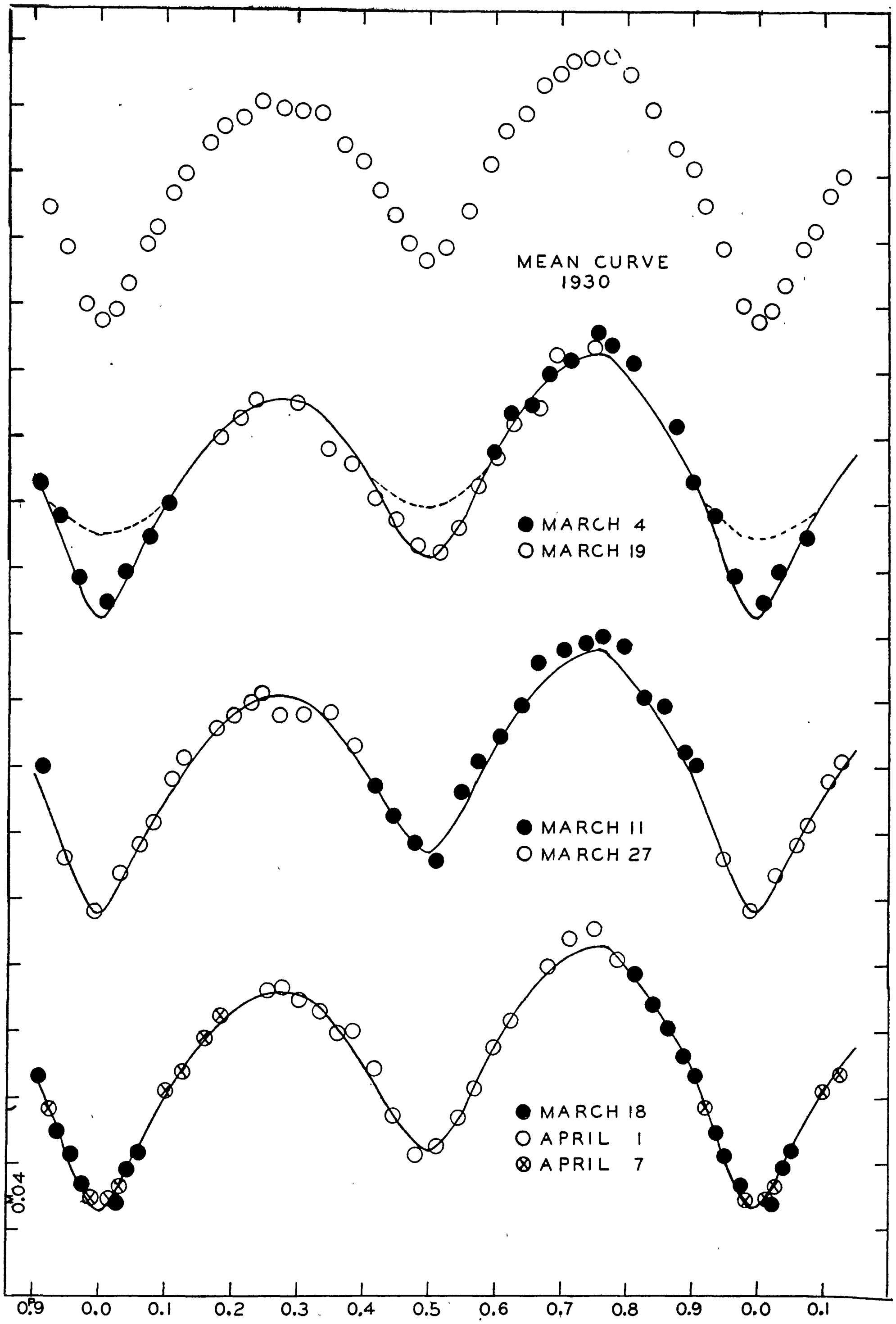

FIG. 1.—Light-curves of 44 i Bootis for 1930

After rectification, the Washburn light-curve shows two unequal minima with light-losses of $1 - \lambda_1 = 0.124$ and $1 - \lambda_2 = 0.088$. After a coefficient of limb darkening of 0.8 was assumed for each component, solutions were made with several values for k, the ratio of radii. Plotting the sums of the squares of the mean deviations of the normals from each of the computed curves revealed a range of indeterminancy of k between two values of 0.9, with the larger and smaller star alternately being also the brighter. For values of k outside this range, the computed curves leave relatively larger residuals from the normals of Table 3. The end-values of the ratio of radii in the apparent range of indeterminancy yield luminosity ratios of 0.55 and 0.80. From inspection of the intensity of the spectral lines, Popper[10] adopts the luminosity ratio 0.6 and states that the true value certainly lies between 0.5 and 0.85. Also from intensity measures in the spectrum, Petrie[11] finds this ratio to be 0.94 ± 0.04.

We have computed the photometric elements in Table 4 from the extreme values for k derived above, as well as for the intermediate assumption of equally large components. The absolute dimensions are based on Popper's spectroscopic elements. The elements in Table 4 must be regarded as but crude approximations not only because of the indeterminancy inherent in such shallow eclipses but also because of probable inadequacy of the simple theory used to derive them. The points of greatest interest in Table 4 are the near-contact of the two stars and the incompatibility of the mass and luminosity ratios with the standard mass-luminosity relation.

TABLE 4

ELEMENTS OF 44 i BOOTIS

Ratio of radii	k	0.90	1.00	1.11
Fractional luminosity of component 1	L_1	0.56	0.58	0.64
Fractional luminosity of component 2	L_2	0.44	0.42	0.36
Semi-major axis (1)	a_1	0.36	0.38	0.38
Intermediate semi-axis (1)	b_1	0.35	0.37	0.37
Polar semi-axis (1)	c_1	0.34	0.35	0.35
Semi-major axis (2)	a_2	0.41	0.38	0.34
Intermediate semi-axis (2)	b_2	0.34	0.33	0.31
Polar semi-axis (2)	c_2	0.27	0.30	0.29
Inclination of the orbital plane	i	61°6	63°6	63°6
Mass ratio	m_2/m_1	0.50	0.50	0.50
Mean radius in terms of the sun (1)	R_1	0.66	0.71	0.70
Mean radius in terms of the sun (2)	R_2	0.64	0.68	0.63
Mass in terms of the solar mass (1)	m_1	1.0	1.0	1.0
Mass in terms of the solar mass (2)	m_2	0.5	0.5	0.5
Assumed coefficient of darkening (1 and 2)	u	0.8	0.8	0.8

Also of interest is the difference in the magnitude of the system at the two maxima. In Figure 2 are plotted the photoelectric observations by Nikonov at both 8000 A and 4250 A, and the photoelectric observations of Shapley and Calder. The zero-point of the solid curve, representing the Washburn observations, has been adjusted to force agreement of the secondary maximum with the other curves. This adjustment also gives agreement at both minima, whereas the primary maximum falls 0.03 mag. too low. The reality of this phenomenon is demonstrated in Figure 1, where observations made on individual nights are plotted. The solid curves in Figure 1 represent the mean curve of Table 3. In the discussion of the Harvard curves of 1935–1936, Shapley and Calder found:[8] "In 1935 the first maximum was slightly, but most certainly higher than the secondary maximum;

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[10] *Ap. J.*, 97, 394, 1943.

[11] Presented in a paper at the seventy-sixth meeting of the American Astronomical Society and kindly communicated by the author in advance of publication.

in 1936 the reverse may have been the situation but the observations are too meager to justify stressing this evidence of variation in the light-curve." They also found the primary minimum 0.01 mag. deeper and the secondary 0.02 mag. shallower in 1936 than in 1935. The observations plotted in Figure 2 represent their mean curve. Not shown in Figure 2 is the excellent photographic curve determined by Plaut, who found equal maxima and amplitudes of minima agreeing well with the photoelectric results. Popper's spectroscopic results indicate a circular orbit. The Washburn and 1935 Harvard curves give $e \cos \omega = 0.00$. From much more meager material the 1936 Harvard curve gives $e \cos \omega = -0.02$. Nikonov's curves, which are of relatively low weight for the determination of times of minima, indicate $e \cos \omega = 0.0$ within their rather large probable error. Although the presence of a small amount of eccentricity is not ruled out, an attempt to account for the irregularities in the height of the maxima of the light-curves by means

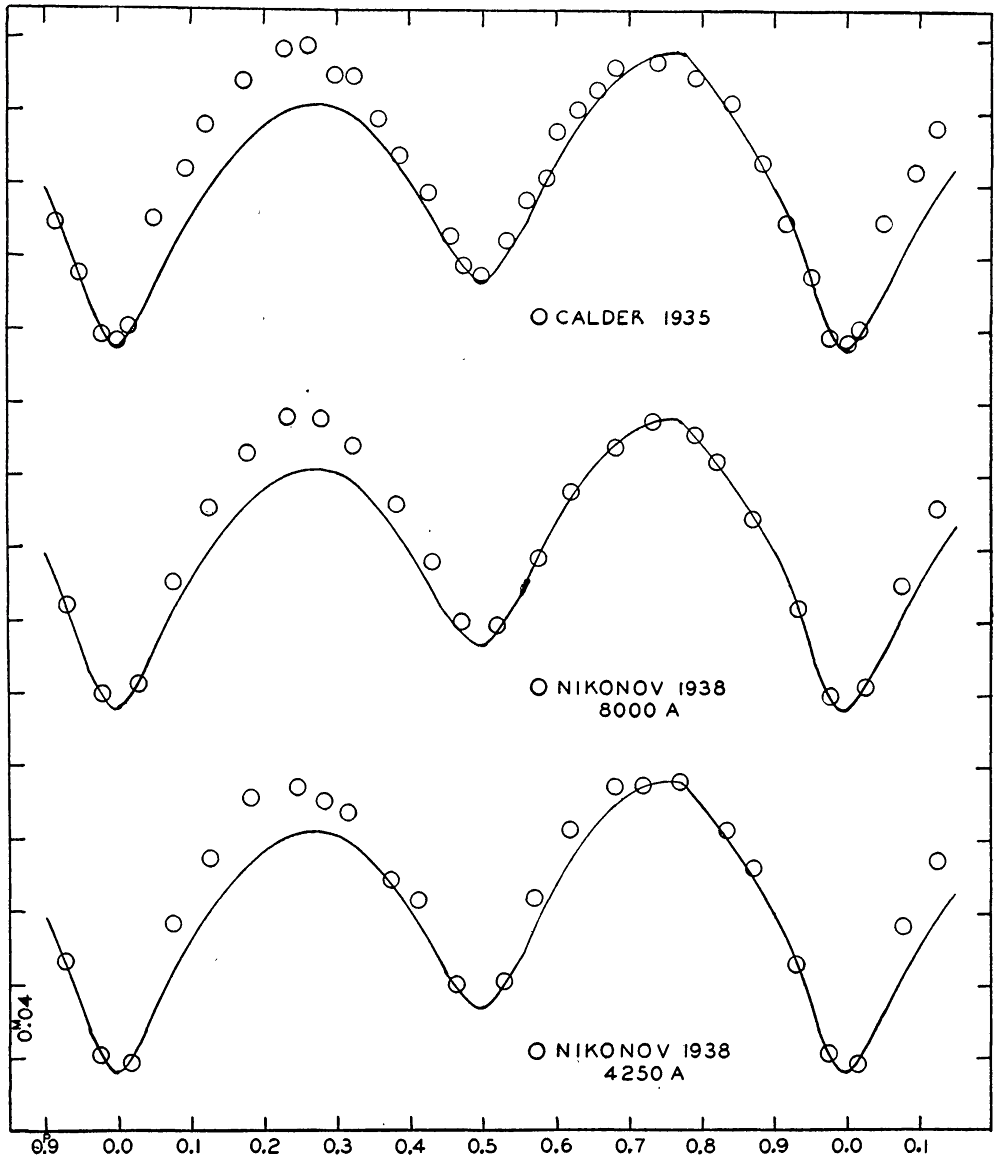

FIG. 2.—Observed light-curves of 44 i Bootis

of an apsidal motion does not appear feasible with the presently available material. However, it may be recalled that very similar asymmetries have been observed for β Lyrae. Stebbins,[12] using a rubidium cell, found the maximum following the primary minimum of this star to be 0.02 mag. lower than the other maximum, and Huffer,[13] with a potassium cell, found the maxima of equal brightness. Moreover, Smart,[14] also using a potassium cell, agreed with Stebbins in finding the first maximum about 0.02 mag. fainter. In an attempt to explain these asymmetries, Kuiper[15] has formulated a theory of "contact binaries," that is, binaries with common atmospheric envelopes. From this theory he concludes: (1) Contact binaries with unequal components are unstable; (2) matter streams from the more massive to the less massive component as long as the masses are unequal; and (3) the difference in magnitude between the components is abnormally large for the mass ratio. The elements in Table 4 show that the nearly equal photometric radii of the components each approximate 0.4 of the distance between centers. Moreover, our neglect of such effects as the gravity-darkening, which should be appreciable in this system, will produce radii which are an underestimation of the true values. Indeed, this fact is undoubtedly a partial cause of the abnormally high densities found for the W Ursae Majoris stars in general. It is possible, therefore, that the eclipsing system of 44 i Bootis constitutes a "contact binary" in which the equalization of the masses by the streaming of matter from the more massive to the least massive is still in process, the masses at the present time being in the ratio of 2 to 1. However, the difficulty remains that the ratio of luminosities inferred from the depths of the eclipses, as well as from the ratio of intensities of the spectral lines of the two components, is abnormally *small* for a mass ratio of 2—an effect just contrary to that expected from the theory of contact binaries as formulated by Kuiper.

The absolute visual magnitude of the brighter component has been determined, spectroscopically, at Victoria,[16] Mount Wilson,[17] and the Norman Lockyer[18] observatories. The mean value is $M_v = +4.63 \pm 0.13$. Taking Kuiper's value,[19] 0.63 mag., for the magnitude difference between the visual components at maximum of the eclipsing system in visual light, we then find the absolute visual magnitude of the eclipsing system, at mean maximum, to be +5.26 mag. The resulting spectroscopic parallax is $0\overset{''}{.}073$, as compared with the trigonometric value of $0\overset{''}{.}078 \pm 0\overset{''}{.}008$.[19] If we designate the brighter visual companion by 1 and the brighter and fainter components of the eclipsing system by 2 and 3, respectively, then, from the elements of the visual orbit as determined by Strand, we find the total mass of the system, $m(1 + 2 + 3) = 2.53\odot$. From Table 4 we have $m_2 = 1.0\odot$ and $m_3 = 0.5\odot$, leaving $m_1 = 1.0\odot$.

An inspection of Figures 1 and 2 shows an apparent deformation of the maximum near phase $0\overset{P}{.}3$. A search through the literature of the W Ursae Majoris stars revealed similar distortions in most well-determined light-curves of variables of this type.[20] In an attempt to test the reality of this phenomenon, a new series of observations of 44 i Bootis was undertaken in 1947 with the photomultiplier photometer, constructed by Dr. Whitford and used in connection with a recording milliameter permitting continuous recording of the light-changes. These observations, made on 7 nights of excellent sky transparency, are presented in Figure 3. The solid curves in Figure 3 represent the mean curve from the 1930 observations. The star HR 5581, described in Table 1, was used for comparison purposes, with additional check measures made on HR 5635. The observa-

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[12] *Lick Obs. Bull.*, **8**, 186, 1915.

[13] *Pub. Washburn Obs.*, **15**, 209, 1931.

[14] *M.N.*, **95**, 647, 1935.

[15] *Ap. J.*, **93**, 29, 1941.

[16] R. K. Young and W. E. Harper, *Pub. Dom. Ap. Obs.*, **3**, 3, 1927.

[17] W. S. Adams *et al.*, *Mt. W. Contr.*, No. 511; *Ap. J.*, **81**, 58, 1935.

[18] W. B. Rimmer, *Mem. R.A.S.*, **62**, 113, 1923.

[19] *Ap. J.*, **88**, 429, 1938.

[20] E.g., E. Woodward, *Harvard Circ.*, No. 446, 1942.

tions of the comparison stars are grouped in Table 5, and it is evident that, as in 1930, they may be considered constant. The observational procedure was to make 1- to 2-minute tracings on the variable star, alternated with similar exposures to the comparison star, except that longer exposures were made on the variable when rapid changes became evident.

The deformity of the light-curve at maximum light, suggested in the previous results, is fully confirmed by the 1947 observations. Furthermore, this deformity appears to be the source of the apparent variation in the relative intensities at the two maxima dis-

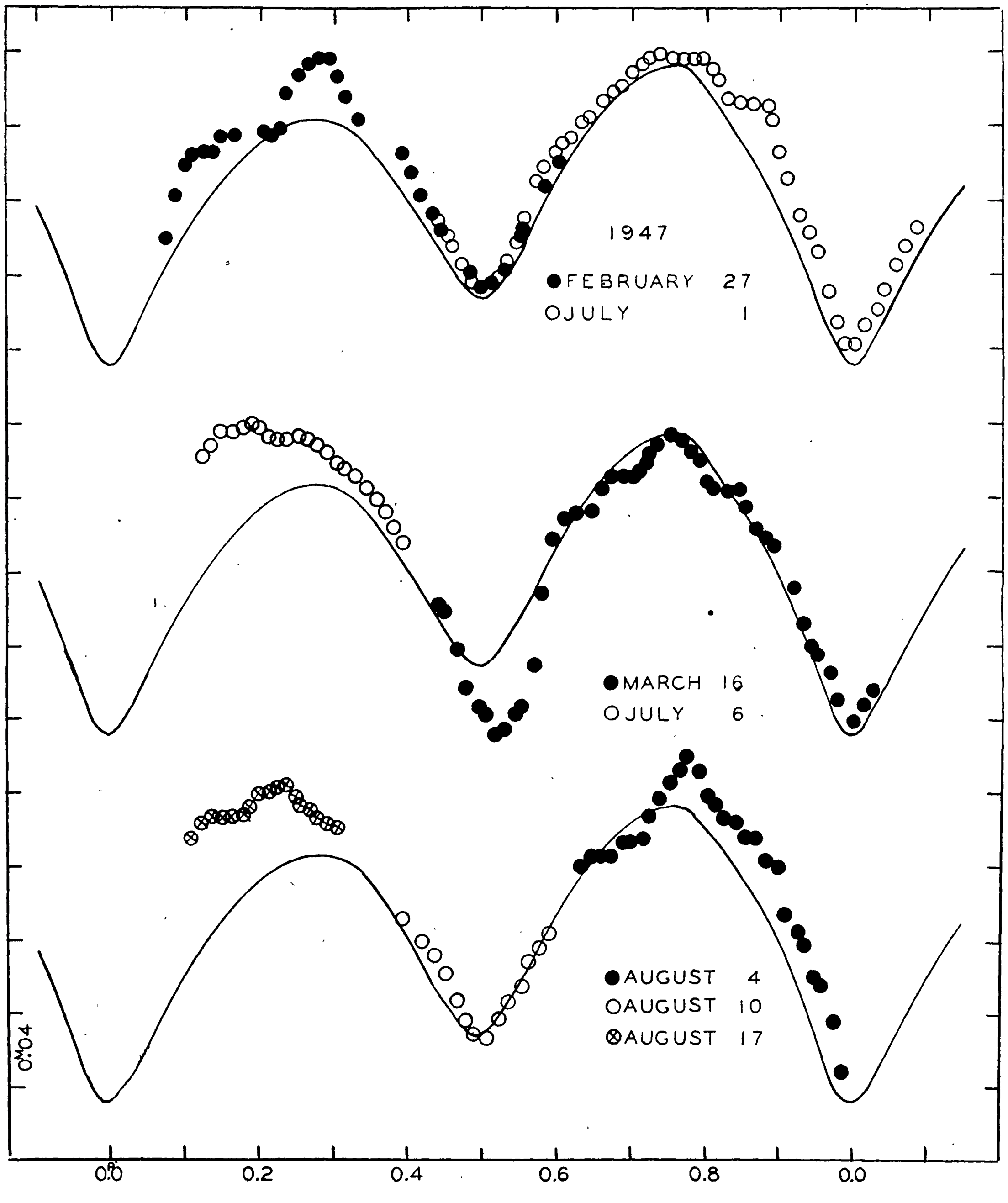

Fig. 3.—Light-curves of 44 i Bootis for 1947

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cussed above. This is especially evident in the long runs made on February 27, March 16, and August 4, 1947, which show a nearly constant, premaximum intensity of the same order of magnitude as the apparently stunted primary maximum observed in 1930.

The explanation of the light-variations of this variable is complicated further by the peculiar secondary minimum observed on March 16. Both minima were observed on this night, and the faintness of the secondary minimum is well established. Furthermore, as is evident from Figure 3 and Table 6, this same secondary minimum is also considerably displaced from the normal $0^{\mathrm{P}}.5$ position of the other secondary minima, whereas the primary minimum, observed the same night, appears to occur at the normal phase.

TABLE 5

CONSTANCY OF THE COMPARISON STARS

JD	5635>5581	Residual	No.
2432368	$0^{\mathrm{m}}.068$	$0^{\mathrm{m}}.000$	10
369	.066	+ .002	8
373	.067	+ .001	6
374	.069	− .001	7
376	.068	.000	10
402	0.069	−0.001	13
Mean	0.068	±0.0008	43

TABLE 6

NEW EPOCHS OF MINIMA OF 44 i BOOTIS

PRIMARY		SECONDARY	
JD	O−C	JD	O−C
2432339.7838	$+0^{\mathrm{d}}.0286$	2432344.8455	$+0^{\mathrm{d}}.0282$
2432369.7778	+ .0280	2432361.7221	+ .0329
2432373.7953	+ .0284	2432362.7892	+ .0288
2432380.7585	+0.0286	2432374.7332	+0.0290
Mean	$+0^{\mathrm{d}}.0284 \pm 0^{\mathrm{d}}.0002$	Mean*	$+0^{\mathrm{d}}.0287 \pm 0^{\mathrm{d}}.0003$

* Omitting the value for March 16, JD 2432361.

Previous investigators of this system have felt that, if the minima were accurately timed to approximately the year 1950, it would be possible to derive the mass function of the visual components from the variation in the period of the eclipsing system. Plaut,[7] using the minima available in 1939, found that $m_1/(m_1 + m_2 + m_3) = 0.4$, identical with the result which we have derived above by other means; but, even after he had corrected the observed epochs of minima for motion in the long-period orbit, using this mass-function, there remained a systematic deviation from a constant period.

Table 6 contains the minima determined in 1947, together with the residuals from the elements in equation (1). The computed epochs of secondary minima are for $e = 0.00$. The well-determined epochs of primary minima which have become available since 1916 are shown in Table 7.

The residuals of Table 7 are plotted in Figure 4. Similar computations were made, using other values for the parallax and mass function; but no combination was found that appreciably changes the curvature shown in Figure 4. Unfortunately, there are no minima available to me between 1939 and 1947; but Popper,[10] in computing the phases for his spectroscopic observations, found not only that the period given by Plaut had run off in the same sense as that found here but that the increase was also evident over the 3-year interval covered by his observations. The epoch of minimum attributed to Popper in Table 7 is obtained from his spectrographic results.

Of the known W Ursae Majoris systems, there are six which have been sufficiently observed to test the constancy of their periods, namely: 44 i Bootis, $P = 0^{d}26$; SW Lacertae,[20] $P = 0^{d}32$; W Ursae Majoris,[20] $P = 0^{d}33$; U Pegasi,[21] $P = 0^{d}37$; and AK Herculis,[20] $P = 0^{d}42$. All six have been found to have varying periods, and such explanations as long-period orbital motion and apsidal motion have been invoked to account for them. Although the sample is small and it may be argued that it requires more observations to prove the constancy, than it does to prove the variability, of a period, it appears that period changes may be a characteristic of this type of system. In the theory of contact binaries proposed by Kuiper, he finds that the period of such a system would be affected by (1) mass transfer from one component to the other; (2) loss of mass and momentum by ejection of matter; and (3) pressure at the interface between the two components. The effects of characteristics 1 and 2 are to decrease the period, and the effect of point 3 is a slow regression of the line of apsides, the period of which is largely dependent upon the degree of contact. Kuiper concludes that the only mechanism available to explain an increase in the period is a sudden decrease in the diameter of the star; but in this case the increase of period would be only temporary.

TABLE 7

MINIMA OF 44 i BOOTIS

1 JD	2 $O-C_1$	3 $O-C_2$	4 $O-C_3$	5 Type and Authority
2421113.2588....	$0^{d}0000$	$0^{d}0000$	$0^{d}0000$	pv; Hertzsprung, *B.A.N.*, No. 165, 1929
23204.4933....	− .0129	− .0157	− .0125	pv; Kuiper, *B.A.N.*, No. 165, 1929
24646.9724....	− .0172	− .0212	− .0135	pg; Schilt, *Ap. J.*, **64**, 215, 1926
25398.4398....	− .0189	− .0244	− .0156	pg; Osterhoff, *B.A.N.*, No. 321, 1939
25733.4699....	− .0180	− .0266	− .0137	pg; Osterhoff, *B.A.N.*, No. 321, 1939
26058.8554....	− .0181	− .0238	− .0146	pe; Huffer, *Pub. A.A.S.*, **6**, 365, 1930
27587.5035....	− .0214	− .0262	− .0169	vs; Plaut, *B.A.N.*, No. 321, 1939
27948.7741....	− .0243	− .0289	− .0198	pe; Calder, *Harvard Bull.*, No. 907, 1938
28635.4351....	− .0238	− .0283	− .0196	pg; Plaut, *B.A.N.*, No. 321, 1939
30152.5752....	− .0178	− .0235	− .0143	sp; Popper, *Ap. J.*, **97**, 407, 1943
32380.7583....	0.0000	0.0000	0.0000	pe; Eggen, present study

NOTES FOR TABLE 7

1. Epoch.
2. Residuals of the uncorrected minima from the linear elements: Minimum = JD 2421113.2588 + $0^{d}26780832\,E$.
3. Residuals from the elements: Minimum = JD 2421113.1486 + $0^{d}26780754\,E$ after the epochs are corrected for motion in the long-period orbit, using Strand's elements and assuming a negative sign for the inclination of the orbital plane. $\pi = 0''073$, and $m_1/(m_1 + m_2 + m_3) = 0.40$.
4. Residuals from the elements: Minimum = JD 2421113.3690 + $0^{d}26780910\,E$ after correcting the minima as in 3, except that the inclination of the orbital plane is here assumed to be positive.
5. Source of minima and method of observation.

In view of these considerations, it does not appear possible at the present time to segregate the change in period of the eclipsing system resulting from the long-period

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[21] E. Woodward and F. Recillas, *A.J.*, **51**, 101, 1944.

orbital motion from those changes due to purely intrinsic causes and in this way to determine the mass function of the visual pair. In fact, the mass function 0.4, determined above in a different way, appears to be well established, and the problem may be reversed. Moreover, because of the presence of these intrinsic changes in the period, we are not able to fix the sign of the inclination in the visual orbit with any great certainty, although from Figure 4 we would select the positive value as being the most likely.

As we have already pointed out, the elements of the eclipsing system of 44 i Bootis, given in Table 4, must be considered as only crude approximations to the true values.

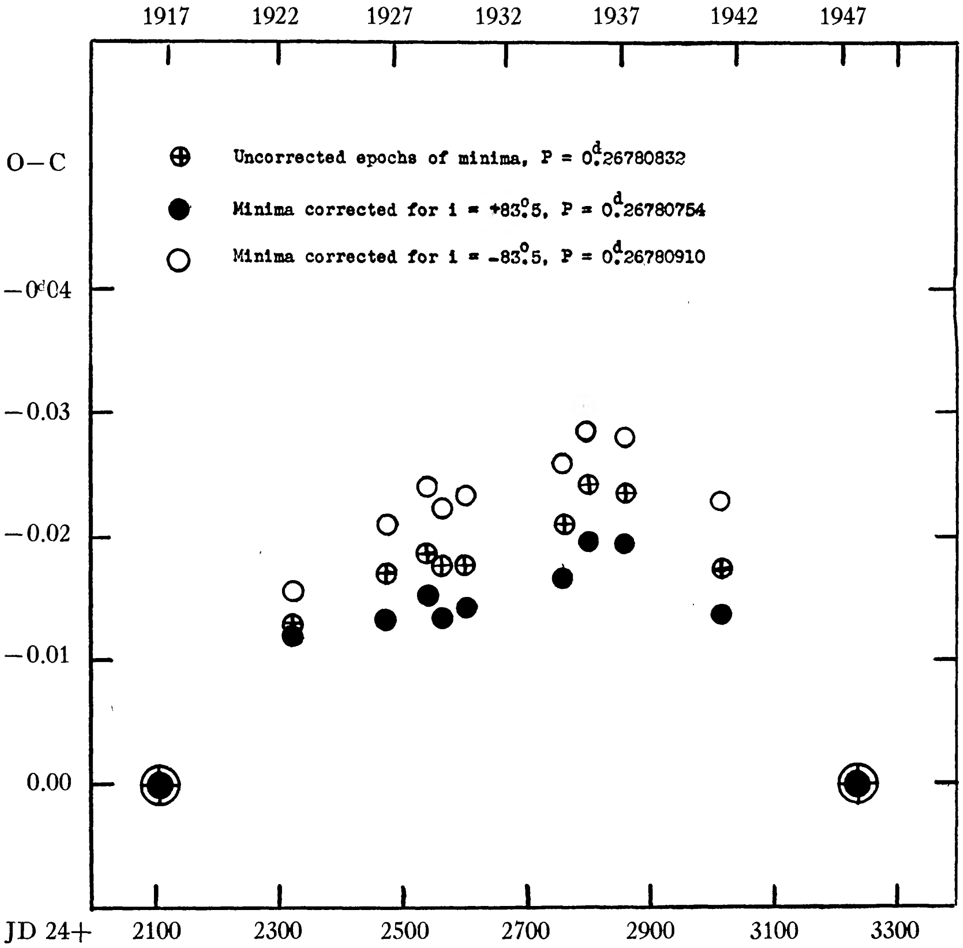

FIG. 4.—Variation of the period of 44 i Bootis

Before more definitive results can be obtained, the simple theory used in deriving these elements must be modified to explain: (1) the deformation of the two maxima of the light-curve; (2) the deepening and the displacement of the secondary minimum observed on March 16, 1947; (3) the intrinsic variation of the period; and (4) the apparent violation of the mass-luminosity relation. There is every evidence that these effects are not characteristic of 44 i Bootis alone but are also to be found in all the W Ursae Majoris systems. The variation of the period of these systems has already been discussed, as has the deformation of maximum light which is evident in many published photographic curves of variables of this type. It might be mentioned that I have already detected this deformation in an unpublished series of observations of W Ursae Majoris itself, made in 1947 in a similar manner to that described here for 44 i Bootis. Also, the apparent violation of the mass-luminosity relation appears to be another characteristic of these variables. In Table 8 are gathered the relevant data for systems which have been subjected to spectroscopic study. From the masses alone, the mass-luminosity relation would require the magnitudes of the components of these systems to differ by 2–3 mag. Moreover, the light-curves show nearly equal minima, and the fact that two spectra are observable at all is a contradiction. Therefore, we must assume that the masses and/or the luminosities are vitiated by some neglected effects or else that the mass-luminosity relation breaks down for the components of these systems. Some of these anomalies might be explained as resulting from the large reflection effect to be expected in such close systems. If we were to assume that the detection of the spectra of the fainter components were possible only because of the light reflected from the close primary stars, then most of the difficulty with the mass-luminosity relation could be explained. If the secondary component appears nearly as bright as the primary, as a result of this large reflection or even of the heating, effect, then the near-equality of the two eclipses is to be expected. Furthermore, the presence of such a large reflection effect would help explain the variation in the relative intensities of the spectral lines observed by Struve[22] and Chang[23] in the systems of YY Eridani and AH Virginis, respectively. The values given in Table 8 for the inclinations of the orbital planes provide another, indirect argument for the presence of a large amount of reflected light within the system. The preponderance of relatively small inclinations would be expected from photometric solutions of light-curves without first removing the excess light. Rather small values for the inclinations are quite common to these systems, those of the well-studied stars mentioned previously in connection with the change in period, for which no spectroscopic data are available, being[24] SW Lacertae, $i = 73°$; VW Cephei, $i = 63°$; U Pegasi, $i = 73°$; and AK Herculis, $i =$

TABLE 8

ELEMENTS OF TYPICAL W URSAE MAJORIS TYPE SYSTEMS

Star	Period	Sp.	i	L	R	m	M_v	Source
44 i Boo 1	$0\overset{\mathrm{d}}{.}27$	G2*	63°	0.58	$0.7\odot$	$1.0\odot$	+4.6	Popper, *Ap. J.*, **97**, 393, 1944
44 i Boo 2	....	G2	.....	.42	0.7	0.5	+5.0	Eggen, present study
YY Eri† 1	.32	G5	.....	....	....	....	....	Struve, *Ap. J.*, **106**, 96, 1947
YY Eri 2	....	G5	.....	....	....	‡	....	
W UMa 1	.33	F8	75	.60	0.7	0.8	....	Adams, *Ap. J.*, **49**, 189, 1919
W UMa 2	....	F8	.....	.40	0.6	0.5	....	Huffer, *Ap. J.*, **79**, 369, 1934
AH Vir† 1	.41	K0	83	.65	1.2	1.4	....	Chang, *Ap. J.*, **107**, 96, 1948
AH Vir 2	....	K0	.....	.35	0.7	0.6	....	
ER Ori 1	.42	G2	71	.52	0.6	0.5	....	Struve, *Pub. A.S.P.*, **58**, 34, 1944
ER Ori 2	....	G2	.....	.48	0.6	0.3	....	
S Ant 1	0.65	A8	62	.67	1.7	0.8	....	Joy, *Ap. J.*, **64**, 293, 1926
S Ant 2	....	A8	.....	0.33	1.3	0.4	....	

* Classification by Joy and quoted by Wyse.

† Struve and Chang have found that the brighter, more massive components of YY Eri and AH Vir are eclipsed at secondary minima; but it is premature to discuss the significance of these results until accurate light-curves of the variables are available.

‡ $m_1 \sin^3 i = 0.72\odot$; $m_2 \sin^3 i = 0.47\odot$.

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[22] *Ap. J.*, **106**, 96, 1947.

[23] *Ap. J.*, **107**, 96, 1948.

[24] E. Woodward, *Harvard Circ.*, No. 447, 1942.

60°. Observations of the light-variations of these systems in several colors appear to be the best test of this hypotheses.

Similarly, observations in several colors should also throw more light on the nature of the deformed maximum in the light-curves of Figure 3. This effect is possibly connected with the presence of a large amount of reflected light, but either nonsynchronism of orbital and rotational periods or nonuniform reflecting surfaces would also be required. In addition, the recent possible detection of "flares" and "spots" on the eclipsing variable AR Lacertae by G. Kron[25] may provide a basis for similar explanation of the anomalies observed in 44 i Bootis.

Therefore, we conclude that spectroscopic and photometric elements of 44 i Bootis in particular and of the W Ursae Majoris stars in general should be regarded with suspicion. On the other hand, further study of these systems should provide much information regarding the evolution of binary systems. Moreover, the shortness of their periods and the presence of exaggerated reflection and ellipticity effects makes these variables excellent proving grounds for the theories to be applied to more normal systems.

I am indebted to Dr. A. E. Whitford, who designed and constructed the photoelectric photometer at the Washburn Observatory. The latest improvements of the photometer were made possible by a grant to Dr. Joel Stebbins from the Gould Fund of the National Academy of Sciences. The debt due Dr. Stebbins not only for making available the 1930 series of observations of 44 i Bootis but also for the benefits of his advice and experience cannot adequately be expressed.

[25] *Pub. A.S.P.*, **59**, 261, 1947.